Exercise No: 7.4

Question 1:

Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A (2, −2) and B(3, 7).

Solution:

If the ratio in which P divides AB is k: 1, then the coordinates of the point P will be ($\frac{kx_2+x_1}{k+1}$, $\frac{ky_2+y_1}{k+1}$).

Let the given line divides the line segment joining the points A(2, −2) and B(3, 7) in a ratio k : 1.

The coordinates of the point of division = $(\frac{3k+2}{k+1}, \frac{7k-2}{k+1})$

This points also lies on $2x + y - 4 = 0$

$2(\frac{3k+2}{k+1}) + (\frac{7k-2}{k+1}) - 4 = 0$

$\frac{6k+4+7k-2-4k-4}{k+1} = 0$

$9k - 2 = 0$

$k = \frac{2}{9}$

Question 2:

Find a relation between $x$and y if the points (x, y), (1, 2) and (7, 0) are collinear.

Solution:

If the given points are collinear, the area of triangle formed by these points will be 0. Area of a triangle = $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Area = $\frac{1}{2}[(2 - 0) + 1(0 - y) + 7(y - 2)]$

$0 = \frac{1}{2}[2x - y + 7y - 14]$

$0 = \frac{1}{2}[2x + 6y - 14]$

$2x + 6y - 14 = 0$

$x + 3y - 7 = 0$

This is the required relation between x and y.

Question 3:

Find the centre of a circle passing through the points (6, −6), (3, −7) and (3, 3).

Solution:

Let O (x, y) be the centre of circle and let (6, −6), (3, −7) and (3, 3) are A, B, and C points on the circumference of circle.

OA = $\sqrt{(x-6)^2+(y+6)^2}$

OB = $\sqrt{(x-3)^2+(y+7)^2}$

OC = $\sqrt{(x-3)^2+(y-3)^2}$

OA = OB (Radius of circle)

$\sqrt{(x-6)^2+(y+6)^2} = \sqrt{(x-3)^2+(y+7)^2}$

$x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 49 + 14$

$-6x - 2y + 14 = 0$

$3x + y = 7$ …….(i)

OA = OC (Radius of circle)

$\sqrt{(x-6)^2+(y+6)^2} = \sqrt{(x-3)^2+(y-3)^2}$

$x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 9 - 6y$

$-6x + 18y + 54 = 0$

$-3x + 9y = -27$ …….(ii)

Adding equation (i) and (ii)

$10y = -20$

$y = -2$

From equation (i)

$3x - 2 = 7$

$3x = 9$

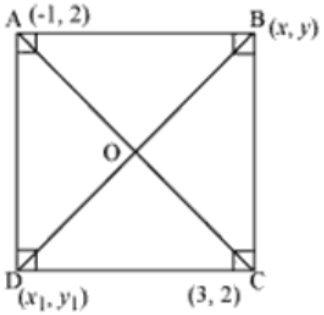
$x = 3$

Hence, the centre of circle is O (3, −2)

Question 4:

The two opposite vertices of a square are (−1, 2) and (3, 2). Find the coordinates of the other two vertices.

Solution:



Let ABCD be a square having (−1, 2), (3, 2) as vertices A and C respectively and (x, y), (x_1, y_1) be the coordinate of vertex B and D respectively.
The sides of a square are equals to each other. Hence,
AB = BC

$\sqrt{(x + 1)^2 + (y - 2)^2} = \sqrt{(x - 3)^2 + (y - 2)^2}$

$x^2 + 2x + 1 + y^2 - 4y + 4 = x^2 + 9 - 6x + y^2 + 4 - 4y$

$8x = 8$

$x = 1$

We know that in a square all interior angles are of 90^o
Hence, in ΔABC,

$AB^2 + BC^2 = AC^2$

$(\sqrt{(1 + 1)^2 + (y - 2)^2})^2 + (\sqrt{(1 - 3)^2 + (y - 2)^2})^2 = (\sqrt{(3 + 1)^2 + (2 - 2)^2})^2$

$4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$

$2y^2 + 16 - 8y = 16$

$2y^2 - 8y = 0$

$(y - 4) = 0$

$y = 0$ or 4

We know that in a square diagonals are of equal length and bisect each other at 90^o.
Let O be the midpoint of AC so it will also be the midpoint of BD.
Coordinate of point O = $(\frac{-1+3}{2}, \frac{2+2}{2})$

$(\frac{1 + x_1}{2}, \frac{y + y_1}{2}) = (1, 2)$

$\frac{1 + x_1}{2} = 1$

$1 + x_1 = 2$

$x_1 = 1$

$\frac{y + y_1}{2} = 2$

$y + y_1 = 4$

If $y = 0$, $y_1 = 4$
If $y = 4$, $y_1 = 0$

So, Coordinates of other vertices are (1, 0) (1, 4).

Question 5.

The Class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Gulmohar is planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the figure the students are to sow seeds of flowering plants on the remaining area of the plot.
(i) Taking A as origin, find the coordinates of the vertices of the triangle.
(ii) What will be the coordinates of the vertices of Δ PQR if C is the origin? Also calculate the areas of the triangles in these cases. What do you observe?

Solution:

(i)
Taking A as origin, we will take AD as x axis and AB as y axis. Now we may observe that coordinates of points P, Q and R are (4, 6), (3, 2), (6, 5)

Area of triangle $= \frac{1}{2}\ \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

$= \frac{1}{2}\ [4(2 - 5) + 3(5 - 6) + 6(6 - 2)]$

$= \frac{1}{2}\ [-12 - 3 + 24]$

$= \frac{9}{2}$ square units

(ii)
Taking C as origin and CB as x −axis and CD as y −axis the coordinates of vertices P, Q, R are (12, 2), (13, 6), (10, 3).

Area of triangle $= \frac{1}{2}\ \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

$= \frac{1}{2}\ [12(6 - 3) + 13(3 - 2) + 10(2 - 6)]$

$= \frac{1}{2}\ [36 + 13 - 40]$

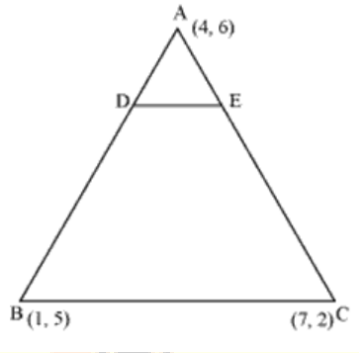
$= \frac{9}{2}$ square units

Area of the triangle is same in both in the cases.

Question 6.

The vertices of a ΔABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of Δ ABC.

Solution:



Given that:

$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$

$\frac{AD}{AD + BD} = \frac{AE}{AE + EC} = \frac{1}{4}$

$\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{3}$

So, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1: 3.

Coordinates of the point P(x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2,y_2) internally in the ratio $m_1 : m_2$ are

$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$,

Coordinates of Point D = $\left(\frac{1\times1+3\times4}{1+3}, \frac{1\times5+3\times6}{1+3}\right)$

= $\left(\frac{13}{4}, \frac{23}{4}\right)$

Coordinates of point E = $\left(\frac{1\times7+3\times4}{1+3}, \frac{1\times2+3\times6}{1+3}\right)$

= $\left(\frac{19}{4}, \frac{20}{4}\right)$

Area of a triangle = $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Area of ΔADE = $\frac{1}{2}\left(4\left(\frac{23}{4} - \frac{20}{4}\right) + \frac{13}{4}\left(\frac{20}{4} - 6\right) + \frac{19}{4}\left(6 - \frac{23}{4}\right)\right)$

= $\frac{1}{2}\left(3 - \frac{13}{4} + \frac{19}{16}\right) = \frac{1}{2}\left(\frac{48-52+19}{16}\right)$

$= \frac{15}{32}$square units

Area of $\Delta ABC = \frac{1}{2}[4(5-2) + 1(2-6) + 7(6-5)]$

$= \frac{1}{2}[12 - 4 + 7]$

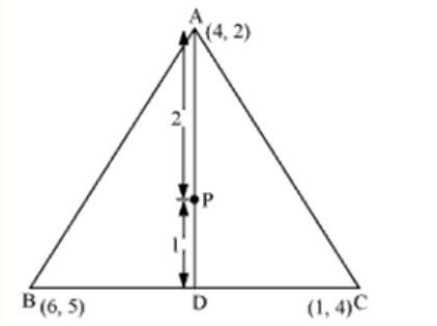
$= \frac{15}{2}$ square units

Clearly the ratio between the areas of ΔADE and of ΔABC is 1 : 16.

Question 7:

Let A (4, 2), B(6, 5)and C(1, 4) be the vertices of ΔABC.
(i) The median from A meets BC at D. Find the coordinates of the point D.
(ii) Find the coordinates of the point P on AD such that AP: PD = 2 : 1
(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ: QE = 2: 1 and CR: RF = 2: 1.
(iv) What do you observe?
(v) If A(x_1, y_1), B (x_2, y_2) and C(x_3, y_3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

Solution:



(i)
Median AD of the triangle will divide the side BC in two equal parts.
So, D is the midpoint of side BC.

Coordinates of D = $(\frac{6+1}{2}, \frac{5+4}{2})$

$= (\frac{7}{2}, \frac{9}{2})$

(ii)
Point P divides the side AD in a ratio 2 : 1.

Coordinates of P = $(\frac{2\times\frac{7}{2}+1\times4}{2+1}, \frac{2\times\frac{9}{2}+1\times2}{2+1})$

$= (\frac{11}{3}, \frac{11}{3})$

(iii)

Median BE of the triangle will divide the side AC in two equal parts, So E is the midpoint of side AC.

Coordinates of E = $(\frac{4+1}{2}, \frac{2+4}{2})$

= $(\frac{5}{2}, 3)$

(iv)

Point Q divides the side BE in a ratio 2: 1.

Coordinates of Q = $(\frac{2\times\frac{5}{2}+1\times6}{2+1}, \frac{2\times3+1\times5}{2+1})$

= $(\frac{11}{3}, \frac{11}{3})$

Median CF of the triangle will divide the side AB in two equal parts.

So, F is the midpoint of side AB.

Coordinates of F = $(\frac{4+6}{2}, \frac{2+5}{2})$

= $(5, \frac{7}{2})$

Point R divides the side CF in a ratio 2 : 1.

(v)

Coordinates of R = $(\frac{2\times5+1\times1}{2+1}, \frac{2\times7\ 2+1\times4}{2+1})$

= $(\frac{11}{3}, \frac{11}{3})$

(vi)

Now we may observe that coordinates of point P, Q, R are same. So, all these are representing same point on the plane i.e. Centroid of the triangle.

(vii)

Now consider a ΔABC having its vertices as A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3). Median AD of the triangle will divide the side BC in two equal parts.

So, D is the midpoint of side BC.

Coordinates of D = $(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2})$

Let centroid of this triangle is O.

Point O divides the side AD in a ratio 2: 1.

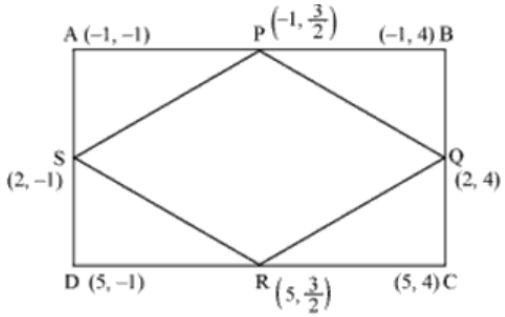
Coordinate of O = $(\frac{2\times\frac{x_2+x_3}{2}+1\times x_1}{2+1}, \frac{2\times\frac{y_2+y_3}{2}1\times y_1}{2+1})$

= $(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3})$

Question 8.

ABCD is a rectangle formed by the points A(−1, −1), B(−1, 4), C(5, 4) and D(5, −1). P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? a rectangle? Or a rhombus? Justify your answer.

Solution:



Length of PQ = $\sqrt{\left(-1-2\right)^2+\left(\frac{3}{2}-4\right)^2}$

$= \sqrt{9+\frac{25}{4}}$

$= \sqrt{\frac{61}{4}}$

Length of QR = $\sqrt{(2-5)^2+(4-\frac{3}{2})^2}$

$= \sqrt{9+\frac{25}{4}}$

$= \sqrt{\frac{61}{4}}$

Length of RS = $\sqrt{(5-2)^2+(\frac{3}{2}+1)^2}$

$= \sqrt{9+\frac{25}{4}}$

$= \sqrt{\frac{61}{4}}$

Length of SP = $\sqrt{(2+1)^2+(-1-\frac{3}{2})^2}$

$= \sqrt{9+\frac{25}{4}}$

$= \sqrt{\frac{61}{4}}$

Length of PR = $\sqrt{(-1 - 5)^2 + (\frac{3}{2} - \frac{3}{2})^2}$

= 6

Length of QS = $\sqrt{(2 - 2)^2 + (4 + 1)^2}$

= 5

Here, all sides of given quadrilateral is of same measure but the diagonals are of different lengths. So, quadrilateral PQRS is a rhombus.