Exercise No: 7.3

Question 1:

Find the area of the triangle whose vertices are:
(i) (2, 3), (−1, 0), (2, −4)
(ii) (−5, −1), (3, −5), (5, 2)

Solution:

(i)
Vertices of triangle: (2, 3), (−1, 0), (2, −4)
Area of a triangle = $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$
Substitute the values,
Area of given triangle = $\frac{1}{2}[2\{0 - (-4)\} + (-1)\{(-4) - (3)\} + 2(3 - 0)]$

$= \frac{1}{2}\{8 + 7 + 6\}$

$= \frac{21}{2}$ square units

(ii)
Vertices of triangle: (−5, −1), (3, −5), (5, 2)
Area of a triangle = $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$
Substitute the values,
Area of given triangle = $\frac{1}{2}[(-5)\{(-5)— (2)\} + 3(2 - (-1)) + 5\{-1 - (-5)\}]$

$= \frac{1}{2}\{35 + 9 + 20\}$

$= \frac{64}{2}$

= 32 square units

Question 2:

In each of the following find the value of 'k', for which the points are collinear
(i) (7, −2), (5, 1), (3, k)
(ii) (8, 1), (k, −4), (2, −5)

Solution:

(i)
For collinear points, area of triangle formed by them is zero.
Therefore, for points A (7, −2), B (5, 1) and C (3, k), Area of triangle ABC= 0

That means, $\frac{1}{2}\ \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = 0$

$\frac{1}{2}\ [7\{1 - k\} + 5\{k - (-2)\} + 3\{(-2) - 1\}] = 0$

$7 - 7k + 5k + 10 - 9 = 0$

$-2k + 8 = 0$

$k = 4$

(ii)

For collinear points, area of triangle formed by them is zero.

Therefore, for points (8, 1), $(kk, -4)$, (2, −5),

Area of triangle= 0

That means,

$\frac{1}{2}\ \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = 0$

$\frac{1}{2}\ [8\{-4 - (-5)\} + \{(-5) - (1)\} + 2\{1 - (-4)\}] = 0$

$8 - 6k + 10 = 0$

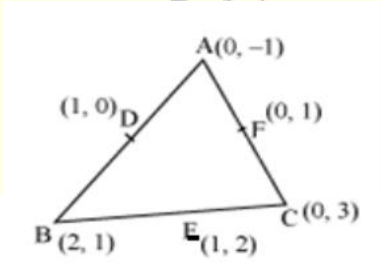
$6k = 18$

$k = 3$

Question 3

Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, −1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Solution:



Let vertices of the triangle be A (0, −1), B (2, 1), C (0, 3) Let D, E, F are midpoints of the sides of this triangle.

Coordinates of D $=(\frac{x1+x2}{2}, \frac{y1+y2}{2})$

$= (\frac{0+2}{2}, \frac{-1+1}{2}) = (1, 0)$

Coordinates of E $= (\frac{2+0}{2}, \frac{1+3}{2}) = (1, 2)$

Coordinates of F $= (\frac{0+0}{2}, \frac{3-1}{2}) = (0, 1)$

Area of a triangle = $\frac{1}{2}$ $\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Area of ΔDEF = $\frac{1}{2}$ $\{1(2 - 1) + 1(1 - 0) + 0(0 - 2)\}$

= 1 2 (1 + 1)

= 1 square units

Area of ΔABC = $\frac{1}{2}$ $[0(1 - 3) + 2\{3 - (-1)\} + 0(-1 - 1)]$

= $\frac{1}{2}$ $\{8\}$

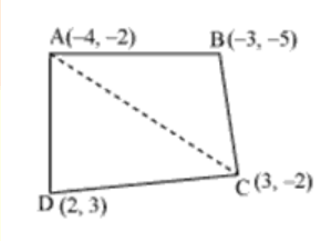
= 4 square units

Therefore, the required ratio = 1: 4

Question 4:

Find the area of the quadrilateral whose vertices, taken in order, are (−4, −2), (−3, −5), (3, −2) and (2, 3).

Solution:



Let vertices of the quadrilateral be A(−4, −2), B(−3, −5), C(3, −2) and D(2, 3). Also, join AC to form two triangles, ΔABC and ΔACD

Area of a triangle = $\frac{1}{2}$ $\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Substituting the values,

Area of ΔABC = $\frac{1}{2}$ $[(-4)\{(-5) - (-2)\} + (-3)\{(-2) - (-2)\} + 3\{(-2) - (-5)\}]$

= $\frac{1}{2}$ (12 + 0 + 9)

= $\frac{21}{2}$ square units

Substituting the values,

Area of ΔACD = $\frac{1}{2}$ $[(-4)\{(-2) - (3)\} + 3\{(3) - (-2)\} + 2\{(-2) - (-2)\}]$

= $\frac{1}{2}$ $\{20 + 15 + 0\}$

= $\frac{35}{2}$ square units

Area of quadrilateral ABCD = area of ΔABC + area of ΔACD

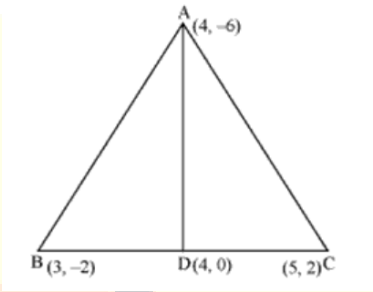
Area of quadrilateral ABCD = $\frac{21}{2} + \frac{35}{2}$

= 28 square units

Question 5:

You have studied in Class IX, (Chapter 9, Example 3), that a median of a triangle divides it into two triangles of equal areas. Verify this result for Δ ABC whose vertices are A (4, −6), B (3, −2) and C (5, 2)

Solution:



Let vertices of the triangle be A (4, − 6), B (3, −2), C (5, 2) Let D be the midpoint of side BC of ΔABC.

Therefore, AD is the median in ΔABC.

Coordinates of point D = $(\frac{3+5}{2}, \frac{-2+2}{2})$

= (4,0)

Area of a triangle = $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Area of ΔABD = $\frac{1}{2}$ [(4){(−2) − (0)} + (3){(0) − (−6)} + (4){(−6) − (−2)}]

= $\frac{1}{2}$ (−8 + 18 − 16)

= −3 square units

But, area cannot be negative.

Hence, the area of ΔABD is 3 square units.

Area of ΔADC = $\frac{1}{2}$ [(4){0 − (2)} + (4){(2) − (−6)} + (5){(−6) − (0)}]

= $\frac{1}{2}$ (−8 + 32 − 30)

= −3 square units

But, area cannot be negative.

Hence, area of ΔADC is 3 square units.

Therefore, clearly median AD has divided ΔABC in two triangles of equal areas.