Exercise 7.1

Question 1:

Find the distance between the following pairs of points:

(i) (2, 3), (4, 1)
(ii) (−5, 7), (−1, 3)
(iii) (a, b), (−a, −b)

Solution:

(i)
We know that the distance between two points (x_1, y_1) and (x_2, y_2) is given by,
$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ Or $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Hence, the distance between (2, 3) and (4, 1) is given by,
Distance = $\sqrt{(2 - 4)^2 + (3 - 1)^2}$
$= \sqrt{(-2)^2 + (2)^2}$
$= \sqrt{4 + 4} = \sqrt{8}$
$= 2\sqrt{2}$

(ii)
We know that the distance between two points (x_1, y_1) and (x_2, y_2) is given by,
$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Hence, the distance between (−5, 7) and (−1, 3) is given by,
Distance = $\sqrt{(-5 - (-1))^2 + (7 - 3)^2}$
$= \sqrt{(-4)^2 + (4)^2}$
$= \sqrt{16 + 16}$
$= \sqrt{32}$
$= 4\sqrt{2}$

(iii)
We know that the distance between two points (x_1, y_1) and (x_2, y_2) is given by, $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Hence, the distance between (a, b) and (−a, −b) is given by,
Distance = $\sqrt{(a - (-a))^2 + (b - (-b))^2}$

$= \sqrt{(2a)^2 + (2b)^2}$

$= \sqrt{4a^2 + 4b^2}$

$= 2\sqrt{a^2 + b^2}$

Question 2.

Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

Solution:

We know that the distance between two points (x_1, y_1) and (x_1, y_1) is given by,

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Now, the distance between the points A(0, 0) and *B*(36, 15) is given by,

$AB = \sqrt{(36 - 0)^2 + (15 - 0)^2}$

$= \sqrt{36^2 + 15^2}$

$AB = \sqrt{1296 + 225}$

$= \sqrt{1521}$

$= 39$

Find distance between two towns A & B in section 7.2

It is given that B is located 36 km east and 15 km north of town A.

Let us take A as origin Therefore, A will be A (0, 0) & B will be B (36, 15)

Since, we have already calculated above the distance between the points (0, 0) and (36, 15).

Hence, we can say that the distance between the two towns A and B discussed in section 7.2 is 39 km.

Question 3:

Determine if the points (1, 5), (2, 3) and (−2, −11) are collinear.

Solution:

Three points are collinear if they lie on a same line i.e., one point lies in between the line joining any other two points.

Let A = (1, 5), B = (2, 3), C = (−2, −11)

We know the distance formula: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Substituting the values,

$AB = \sqrt{(1 - 2)^2 + (5 - 3)^2}$

$= \sqrt{5}$

BC = $\sqrt{(2 - (-2))^2 + (3 - (-11))^2}$

$= \sqrt{4^2 + 14^2}$

$= \sqrt{16 + 196}$

$= \sqrt{212}$

CA = $\sqrt{\left(1 - (-2)\right)^2 + \left(5 - (-11)\right)^2}$

$= \sqrt{3^2 + 16^2}$

$= \sqrt{9 + 256}$

$= \sqrt{265}$

Here, the sum of the distances between any two pairs of points is not equal to the distance between the third pair of points. i.e.,

AB + BC = $\sqrt{5} + \sqrt{212} \neq \sqrt{265}$ = AC

Similarly, AB + AC ≠ BC and AC + BC ≠ AB.

Hence, the given three points (1, 5), (2, 3) and (−2, −11) are not collinear.

Question 4.

Check whether (5, −2), (6, 4) and (7, −2) are the vertices of an isosceles triangle.

Solution:

Let A = (5, −2), B = (6, 4), C = (7, −2)

We know the distance formula: $\sqrt{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2}$

AB = $\sqrt{(5 - 6)^2 + (-2 - 4)^2}$

$= \sqrt{(-1)^2 + (-6)^2}$

$= \sqrt{1 + 36} = \sqrt{37}$

BC = $\sqrt{(6 - 7)^2 + (4 - (-2))^2}$

$= \sqrt{(-1)^2 + (6)^2}$

$= \sqrt{1 + 36} = \sqrt{37}$

CA = $\sqrt{(5 - 7)^2 + (-2 - (-2))^2}$

$= \sqrt{(-2)^2 + 0^2}$

= 2

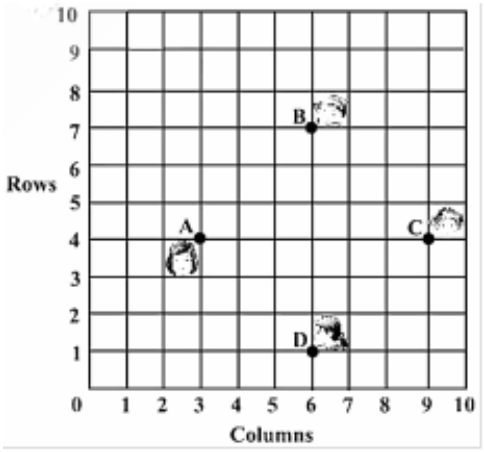
So, AB = BC

Since the two sides are equal in length, therefore, ABC is an isosceles triangle.

Question 5:

In a classroom, 4 friends are seated at the points A, B, C and D as shown in figure. Champa and Chameli walk into the class and after observing for a

few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.



Solution:

From the figure, coordinates of point A, B, C and D are
A = (3, 4), B = (6, 7), C = (9, 4), D = (6, 1)

Distance formula is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

AB = $\sqrt{(3 - 6)^2 + (4 - 7)^2}$
= $\sqrt{(-3)^2 + (-3)^2}$
= $\sqrt{9 + 9}$
= $\sqrt{18}$
= $3\sqrt{2}$
BC = $\sqrt{(6 - 9)^2 + (7 - 4)^2}$
= $\sqrt{(-3)^2 + (3)^2}$
= $\sqrt{9 + 9}$
= $\sqrt{18}$
= $3\sqrt{2}$
CD = $\sqrt{(9 - 6)^2 + (4 - 1)^2}$
= $\sqrt{(3)^2 + (3)^2}$
= $\sqrt{9 + 9}$
= $\sqrt{18}$
= $3\sqrt{2}$
AD = $\sqrt{(3 - 6)^2 + (4 - 1)^2}$
= $\sqrt{(-3)^2 + (3)^2}$
= $\sqrt{9 + 9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

All the sides of quadrilateral are equal, so it may be a square or rhombus on the basis of its diagonal.

Diagonal AC $= \sqrt{(3 - 9)^2 + (4 - 4)^2}$

$= \sqrt{(-6)^2 + 0^2}$

$= 6$

Diagonal BD $= \sqrt{(6 - 6)^2 + (7 - 1)^2}$

$= \sqrt{0^2 + (6)^2}$

$= 6$

Here, all four sides of this figure are of equal length and also two diagonals are of equal length. Therefore, ABCD is a square and thus Champa is correct.

Question 6:

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (−1, −2), (1, 0), (−1, 2), (−3, 0)

(ii) (−3, 5), (3, 1), (0, 3), (−1, −4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Solution:

(i)

Let, A = (−1, −2), B = (1, 0), C = (−1, 2), D = (−3, 0)

Distance formula is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

AB $= \sqrt{(-1 - 1)^2 + (-2 - 0)^2}$

$= \sqrt{(-2)^2 + (-2)^2}$

$= \sqrt{4 + 4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

BC $= \sqrt{(1 - (-1))^2 + (0 - 2)^2}$

$= \sqrt{(2)^2 + (-2)^2}$

$= \sqrt{4 + 4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

CD = $\sqrt{(-1 - (-3))^2 + (2 - 0)^2}$

$= \sqrt{(2)^2 + (2)^2}$

$= \sqrt{4 + 4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

AD = $\sqrt{(-1 - (-3))^2 + (-2 - 0)^2}$

$= \sqrt{(2)^2 + (-2)^2}$

$= \sqrt{4 + 4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

All the sides of quadrilateral are equal, so it may be a square or rhombus on the basis of its diagonal.

Diagonal AC = $\sqrt{(-1 - (-1))^2 + (-2 - 2)^2}$

$= \sqrt{0^2 + (-4)^2}$

$= \sqrt{16}$

$= 4$

Diagonal BD = $\sqrt{(1 - (-3))^2 + (0 - 0)^2}$

$= \sqrt{(4)^2 + 0^2}$

$= \sqrt{16}$

$= 4$

Here, all four sides of this quadrilateral are of equal length and also two diagonals are of equal length.

Therefore, ABCD is a square and the given points are vertices of a square.

(ii)

Let A = (−3, 5), B = (3, 1), C = (0, 3), D = (−1, −4)

Distance formula is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

AB = $\sqrt{(-3 - 3)^2 + (5 - 1)^2}$

$= \sqrt{(-6)^2 + (4)^2}$

$= \sqrt{36 + 16}$

$= \sqrt{52}$

$= 2\sqrt{13}$

BC = $\sqrt{(3-0)^2 + (1-3)^2}$

$= \sqrt{(3)^2 + (-2)^2}$

$= \sqrt{9 + 4}$

$= \sqrt{13}$

CD = $\sqrt{(0 - (-1))^2 + (3 - (-4))^2}$

$= \sqrt{(1)^2 + (7)^2}$

$= \sqrt{1 + 49}$

$= \sqrt{50}$

$= 5\sqrt{2}$

AD = $\sqrt{(-3 - (-1))^2 + (5 - (-4))^2}$

$= \sqrt{(-2)^2 + (9)^2}$

$= \sqrt{4 + 81}$

$= \sqrt{85}$

AC = $\sqrt{(-3 - 0)^2 + (5 - 3)^2}$

$= \sqrt{(-3)^2 + (2)^2}$

$= \sqrt{9 + 4}$

$= \sqrt{13}$

BD = $\sqrt{(3 - (-1))^2 + (1 - (-4))^2}$

$= \sqrt{(4)^2 + (5)^2}$

$= \sqrt{16 + 25}$

$= \sqrt{41}$

We can observe very clearly that, AC + BC = AB, this means that the point C lies on side AB, which implies that the points A, B and C are collinear. Hence, no quadrilateral can be formed with the given points.

(iii)

Let A = (4, 5), B = (7, 6), C = (4, 3), D = (1, 2)

Distance formula is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

AB = $\sqrt{(4 - 7)^2 + (5 - 6)^2}$

$= \sqrt{(-3)^2 + (-1)^2}$
$= \sqrt{9 + 1}$
$= \sqrt{10}$

BC $= \sqrt{(7 - 4)^2 + (6 - 3)^2}$
$= \sqrt{(3)^2 + (3)^2}$
$= \sqrt{9 + 9}$
$= \sqrt{18}$

CD $= \sqrt{(4 - 1)^2 + (3 - 2)^2}$
$= \sqrt{(3)^2 + (1)^2}$
$= \sqrt{9 + 1}$
$= \sqrt{10}$

AD $= \sqrt{(4 - 1)^2 + (5 - 2)^2}$
$= \sqrt{(3)^2 + (3)^2}$
$= \sqrt{9 + 9}$
$= \sqrt{18}$

Diagonal AC $= \sqrt{(4 - 4)^2 + (5 - 3)^2}$
$= \sqrt{0^2 + (2)^2}$
$= \sqrt{0 + 4}$
$= 2$

Diagonal BD $= \sqrt{(7 - 1)^2 + (6 - 2)^2}$
$= \sqrt{(6)^2 + (4)^2}$
$= \sqrt{36 + 16}$
$= \sqrt{52}$
$= 2\sqrt{13}$

We can observe that the opposite sides of this quadrilateral are of equal length, but diagonals are of different lengths. Therefore, ABCD is a parallelogram and given points are vertices of a parallelogram.

Question 7:

Find the point on the x −axis which is equidistant from (2, −5) and (−2, 9).

Solution:

We know that, for a point on the x − axis, its y − coordinate will be 0.

So, let the point on x −axis be $(k, 0)$

Distance between $(k, 0)$ and $(2, -5) = \sqrt{(k - 2)^2 + (0 - (-5))^2}$

$= \sqrt{(k - 2)^2 + (5)^2}$

Distance between $(k, 0)$ and $(-2, 9) = \sqrt{(k - (-2))^2 + (0 - (9))^2}$

$= \sqrt{(k + 2)^2 + (9)^2}$

By given condition, these distances are equal in measure.

$\sqrt{(k - 2)^2 + (5)^2} = \sqrt{(k + 2)^2 + (9)^2}$

$(k - 2)^2 + 25 = (k + 2)^2 + 81$

$k^2 + 4 - 4k + 25 = k^2 + 4 + 4k + 81$

$8k = 25 - 81$

$8k = -56$

$k = -7$

Hence, the required point on the x − axis is $(-7, 0)$.

Question 8:

Find the values of y for which the distance between the points P(2, −3) and Q(10, y) is 10 units.

Solution:

As per the question, it is given that the distance between $(2, -3)$ and $(10, y)$ is 10.

Using distance formula: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Substituting the values, $\sqrt{(2 - 10)^2 + (-3 - y)^2} = 10$

$\sqrt{(-8)^2 + (3 + y)^2} = 10$

Squaring on both the sides,

$64 + (y + 3)^2 = 100$

$(y + 3)^2 = 36$

$y + 3 = \pm 6$

$y + 3 = 6$ or $y + 3 = -6$

Hence, $y = -9, 3$

Question 9:

If Q(0, 1) is equidistant from P(5, −3) and R(x, 6), find the values of x. Also find the distances QR and PR.

Solution:

According to the question, given Q(0, 1) is equidistant from P(5, −3) and R(x, 6) So, PQ = QR

Using distance formula: $\Rightarrow \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\sqrt{(5 - 0)^2 + (-3 - 1)^2} = \sqrt{(0 - x)^2 + (1 - 6)^2}$

$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$

$\sqrt{25 + 16} = \sqrt{x^2 + 25}$

Squaring on both sides,

$25 + 16 = x^2 + 25$

$16 = x^2$

$x = \pm 4$

Hence, point R is (4, 6) or (−4, 6)

Case (i):

When point R is (4, 6)

PR = $\sqrt{(5 - 4)^2 + (-3 - 6)^2}$

= $\sqrt{1^2 + (-9)^2}$

= $\sqrt{1 + 81}$

= $\sqrt{82}$

QR = $\sqrt{(0 - 4)^2 + (1 - 6)^2}$

= $\sqrt{(-4)^2 + (-5)^2}$

= $\sqrt{16 + 25}$

= $\sqrt{41}$

Case (ii):

When point R is (−4, 6)

PR = $\sqrt{(5 - (-4))^2 + (-3 - 6)^2}$

= $\sqrt{(9)^2 + (-9)^2}$

= $\sqrt{81 + 81}$

= $9\sqrt{2}$

QR = $\sqrt{(0 - (-4))^2 + (1 - 6)^2}$

= $\sqrt{(4)^2 + (-5)^2}$

= $\sqrt{16 + 25}$

= $\sqrt{41}$

Hence, $x = \pm 4$,

QR = $\sqrt{41}$,

PR = $\sqrt{82}$, $9\sqrt{2}$

Question 10:

Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (−3, 4).

Solution:

It is given that point (x, y) is equidistant from (3, 6) and (−3, 4)

So, $\sqrt{(x-3)^2+(y-6)^2} = \sqrt{(x-(-3))^2+(y-4)^2}$

$\sqrt{(x-3)^2+(y-6)^2} = \sqrt{(x+3)^2+(y-4)^2}$

Squaring on both sides,

$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$

$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$

$36 - 16 = 6x + 6x + 12y - 8y$

$20 = 12x + 4y$

$3x + y = 5$

Hence, the relation between x and y is $3x + y - 5 = 0$.