Exercise No: 4.4

Question1.

Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Solution:

We know that for a quadratic equation $ax^2 + bx + c = 0$.

Discriminant (D) is $b^2 - 4ac$.

(A) If $b^2 - 4ac > 0$ implies two distinct real roots

(B) If $b^2 - 4ac = 0$ implies two equal real roots

(C) If $b^2 - 4ac < 0$ implies imaginary roots

(i) $2x^2 - 3x + 5 = 0$

Comparing this equation with $ax^2 + bx + c = 0$.

$a = 2$

$b = -3$

$c = 5$

Discriminant (D) $= b^2 - 4ac$

$= (-3)^2 - 4(2)(5)$

$= 9 - 40$

$= -31$

As $b^2 - 4ac < 0$,

Hence, no real root is possible for the given equation.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with $ax^2 + bx + c = 0$.

$a = 3$

$b = -4\sqrt{3}$

$c = 4$

Discriminant $= b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(3)(4)$

$= 48 - 48$

$= 0$

As $b^2 - 4ac = 0$.

So, real roots exist for the given equation and they are equal to each other and the roots will be $-\frac{b}{2a}$ and $-\frac{b}{2a}$.

$-\frac{b}{2a} = \frac{-(-4\sqrt{3})}{2\times3}$

$= \frac{4\sqrt{3}}{6}$

$= \frac{2\sqrt{3}}{3}$

$= 2\sqrt{3}$

Hence, the roots are $2\sqrt{3}$ and $2\sqrt{3}$.

(iii) $2x^2 - 6x + 3 = 0$

Comparing this equation with $ax^2 + bx + c = 0$.

a = 2

b = −6

c = 3

Discriminant = $b^2 - 4ac$

$= (-6)^2 - 4(2)(3)$

$= 36 - 24$

$= 12$

As $b^2 - 4ac > 0$,

So, two distinct real roots exist for this equation as follows:

$x = \frac{-b \pm \sqrt{b2 - 4ac}}{2a}$

$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$

$= \frac{6 \pm \sqrt{12}}{4}$

$= \frac{6 \pm 2\sqrt{3}}{4}$

$= \frac{3 \pm \sqrt{3}}{2}$

Hence, the roots are $\frac{3+\sqrt{3}}{2}$ or $\frac{3-\sqrt{3}}{2}$.

Question2.

Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$

Solution:

We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its Discriminant ($b^2 - 4ac$) will be 0.

(i) $2x^2 + kx + 3 = 0$

Comparing this equation with $ax^2 + bx + c = 0$.

$a = 2$

$b = k$

$c = 3$

Discriminant $= b^2 - 4ac$

$= (k)^2 - 4(2)(3)$

$= k^2 - 24$

For equal roots,

Discriminant = 0

$k^2 - 24 = 0$

$k^2 = 24$

$k^2 = \pm\sqrt{24}$

$= \pm 2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$ or $kx^2 - 2kx + 6 = 0$

Comparing the equation with $ax^2 + bx + c = 0$

$a = k$

$b = -2k$

$c = 6$

Discriminant $= b^2 - 4ac$

$= (-2k)^2 - 4(k)(6)$

$= 4k^2 - 24k$

For equal roots,

$b^2 - 4ac = 0$

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

Either $4k = 0$ or $k - 6 = 0$

$k = 0$ or $k = 6$

If $k = 0$, then the equation will not have the terms 'x^2' and 'x'. Hence, if this quadratic equation has two equal roots, then k should be 6 only.

Question 3.

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.

Solution:

Let the breadth of mango grove be x.

Length of mango grove will be 2x.

Area of mango grove $= (2x)(x)$

$= 2x^2$

Given: $2x^2 = 800$

$x^2 = \frac{800}{2}$

$x^2 = 400$

$x = \pm 20$

Breadth cannot be negative, so the value of x is 20m.

Hence, breadth of mango grove = 20 m and length of mango grove = 2 × 20 = 40 m.

Question 4.

Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Solution:

Let the age of one friend be x years.

Given in question: Age of the other friend will be = (20 − x) years.

Four years ago, age of 1st friend = (x − 4) years

Age of 2nd friend = ((20 – x) − 4)

= (16 − x) years

As per the question,

$(x - 4)(16 - x) = 48$

$16x - 64 - x^2 + 4x = 48$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we get

a = 1

b = −20

c = 112

Discriminant = $b^2 - 4ac$

$= (-20)^2 - 4(1)(112)$

$= 400 - 448$

$= -48$

As $b^2 - 4ac < 0$,

Thus, no real roots are possible for this equation and hence, this situation is not possible.

Question 5.

Is it possible to design a rectangular park of perimeter 80 m and area 400 m²? If so, find its length and breadth.

Solution:

Assume length and breadth of the rectangular park be *l* and b.

Perimeter = 2(l + b) = 80

l + b = 40
b = 40 − l

Area = l × b
= l(40 − l)
= 40l – l^2
Given area = 400
40l – l^2 = 400
l^2 − 40 l + 400 = 0
Comparing this equation with al^2 + bl + c = 0, we get
a = 1
b = −40
c = 400
Discriminant = b^2 – 4ac
= (−40)2 − 4(1)(400)
= 1600 − 1600
= 0
As b^2 – 4ac = 0,
Thus, this equation has equal real roots and hence, this situation is possible. Root of this equation,

$l = -\frac{b}{2a}$ (use this $x = \frac{-b \pm \sqrt{b2 - 4ac}}{2a}$)

$l = -\frac{(-40)}{2(1)}$
$= \frac{40}{2}$
= 20
Length of park, l = 20 m
Breadth of park, b = 40 − l
= 40 − 20
= 20 m