

Exercise No: 4.3

Question1.

Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Solution:

(i)

Given quadratic equation: $2x^2 - 7x + 3 = 0$

$$2x^2 - 7x = -3$$

On dividing both sides of the equation by 2, we get

$$x^2 - \frac{7}{2}x = \frac{-3}{2}$$

$$x^2 - 2 \times x \times \frac{7}{4} = \frac{-3}{2}$$

On adding $\left(\frac{7}{4}\right)^2$ to both sides of equation by, we get

$$(x)^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 = \left(\frac{7}{4}\right)^2 - \frac{3}{2} \quad \left(\text{As } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

$$\left(x - \frac{7}{4}\right)^2 = \frac{49}{16} - \frac{3}{2}$$

$$\left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$$

$$\left(x - \frac{7}{4}\right) = \pm \frac{5}{4}$$

(Cancelling square both the sides)

Either $x - \frac{7}{4} = \frac{5}{4}$ or $x - \frac{7}{4} = -\frac{5}{4}$

$$x = \frac{5}{4} + \frac{7}{4}$$

$$= \frac{12}{4}$$

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

$$\text{When } x - \frac{7}{4} = -\frac{5}{4}$$

$$x = -\frac{5}{4} + \frac{7}{4}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

Hence, the roots of this quadratic equation are 3 and $\frac{1}{2}$.

$$(ii) \quad 2x^2 + x - 4 = 0$$

$$2x^2 + x = 4$$

On dividing both sides of the equation by 2, we get

$$x^2 + \frac{1}{2}x = 2$$

On adding $(\frac{1}{4})^2$ to both sides of the equation, we get

$$(x)^2 + 2 \times x \times \frac{1}{4} + (\frac{1}{4})^2 = 2 + (\frac{1}{4})^2$$

$$(x + \frac{1}{4})^2 = \frac{33}{16}$$

$$x + \frac{1}{4} = \pm \frac{\sqrt{33}}{4}$$

(Cancelling square both the sides)

$$x = \pm \frac{\sqrt{33}}{4} - \frac{1}{4}$$

$$x = \frac{\sqrt{33}-1}{4} \text{ or } \frac{-\sqrt{33}+1}{4}$$

Hence, the roots of this quadratic equation are $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$.

$$(iii) \quad 4x^2 + 4\sqrt{3}x + 3 = 0$$

$$(2x)^2 + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^2 = 0$$

$$(2x + \sqrt{3})^2 = 0$$

$$(2x) + \sqrt{3} = 0 \text{ and } (2x + \sqrt{3}) = 0$$

$$x = \frac{-\sqrt{3}}{2} \text{ and } x = \frac{-\sqrt{3}}{2}$$

Hence, the roots of this quadratic equation are $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$.

$$(iv) \quad 2x^2 + x + 4 = 0$$

$$2x^2 + x = -4$$

On dividing both sides of the equation by 2, we get

$$x^2 + \frac{1}{2}x = -2$$

$$x^2 + 2 \times x \times \frac{1}{4} = -2$$

On adding $(\frac{1}{4})^2$ to both sides of the equation, we get

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

$$(x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 - 2$$

$$\left(x + \frac{1}{4}\right)^2 = \frac{1}{16} - 2$$

$$\left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$$

Since, the square of a number cannot be negative. Thus, there is no real root for the given equation.

Question 2.

Find the roots of the quadratic equations given by applying the quadratic formula.

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Solution:

(i) $2x^2 - 7x + 3 = 0$

On comparing this equation with $ax^2+bx+c=0$, we get

$$a = 2$$

$$b = -7$$

$$c = 3$$

By using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$x = \frac{7 \pm \sqrt{25}}{4}$$

$$x = \frac{7 \pm 5}{4}$$

$$x = \frac{7+5}{4} \text{ or } \frac{7-5}{4}$$

$$x = \frac{12}{4} \text{ or } \frac{2}{4}$$

$$x = 3 \text{ or } \frac{1}{2}$$

Hence, the roots of this quadratic equation are 3 and $\frac{1}{2}$.

(ii) $2x^2 + x - 4 = 0$

On comparing this equation with $ax^2+bx+c=0$, we get

$$a = 2$$

$$b = 1$$

$$c = -4$$

By using quadratic formula, we get

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1+32}}{4}$$

$$x = \frac{-1 \pm \sqrt{33}}{4}$$

$$x = \frac{-1 + \sqrt{33}}{4} \text{ or } \frac{-1 - \sqrt{33}}{4}$$

Hence, the roots of this quadratic equation are $\frac{-1 + \sqrt{33}}{4}$ and $\frac{-1 - \sqrt{33}}{4}$.

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

On comparing this equation $ax^2+bx+c=0$, we get

$$a = 4$$

$$b = 4\sqrt{3}$$

$$c = 3$$

By using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4\sqrt{3} \pm \sqrt{48 - 48}}{2a}$$

$$x = \frac{4\sqrt{3} \pm 0}{8}$$

$$x = -\frac{\sqrt{3}}{2} \text{ or } -\frac{\sqrt{3}}{2}$$

Hence, the roots of this quadratic equation are $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

(v) $2x^2 + x + 4 = 0$

On comparing this equation with $ax^2+bx+c=0$, we get

$$a = 2$$

$$b = 1$$

$$c = 4$$

By using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1-32}}{4}$$

$$x = \frac{-1 \pm \sqrt{-31}}{4}$$

Hence, roots do not exist for this quadratic equation as:

$$D = b^2 - 4ac$$

$$= -31$$

$$b^2 - 4ac < 0$$

hence, the roots of the quadratic equation is not real.

Question 3.

Find the roots of the following equations:

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

(i) $x - \frac{1}{x} = 3, x \neq 0$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Solution:

(i) $x - \frac{1}{x} = 3$

$$x^2 - 1 = 3x$$

$$x^2 - 3x - 1 = 0$$

On comparing this equation with $ax^2 + bx + c = 0$, we get

$$a = 1$$

$$b = -3$$

$$c = -1$$

By using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{9+4}}{2}$$

$$x = \frac{3 \pm \sqrt{13}}{2}$$

Therefore, $x = \frac{3 + \sqrt{13}}{2}$ or $\frac{3 - \sqrt{13}}{2}$.

Roots of quadratic equation are $\frac{3 + \sqrt{13}}{2}$ and $\frac{3 - \sqrt{13}}{2}$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$$\frac{x-7-(x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

$$(x+4)(x-7) = -30$$

$$x^2 - 3x - 28 = -30$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - 2x - x + 2 = 0$$

$$x(x-2) - 1(x-2) = 0$$

$$(x-2)(x-1) = 0$$

$$x = 1 \text{ or } 2$$

Hence, the roots of quadratic equation are 2 and 1.

Question 4.

The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

Solution:

Let Rehman's present age = x years.

His age three years ago = $(x - 3)$ years.

His age five years from now = $(x + 5)$ years.

As per the question, the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$$

$$\frac{2x+2}{(x-3)(x+5)} = \frac{1}{3}$$

$$3(2x + 2) = (x - 3)(x + 5)$$

$$6x + 6 = x^2 + 2x - 15$$

$$x^2 - 4x - 21 = 0$$

$$x^2 - 7x + 3x - 21 = 0$$

$$x(x - 7) + 3(x - 7) = 0$$

$$(x - 7)(x + 3) = 0$$

$$x = 7 \text{ or } x = -3$$

Age cannot be negative. Thus, Rehman's present age is 7 years.

Question 5.

In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Solution:

Let Shefali's marks in Mathematics = x .

Then, her marks in English = $30 - x$.

As per the question, we get

$$(x + 2)(30 - x - 3) = 210$$

$$(x + 2)(27 - x) = 210$$

$$-x^2 + 25x + 54 = 210$$

$$x^2 - 25x + 156 = 0$$

$$x^2 - 12x - 13x + 156 = 0$$

$$x(x + 2) - 13(x - 12) = 0$$

$$(x - 12)(x - 13) = 0$$

$$x = 12 \text{ or } x = 13$$

If the marks in Mathematics is 12, then marks in English will be $30 - 12 = 18$.

If the marks in Mathematics is 13, then marks in English will be $30 - 13 = 17$.

Question 6.

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

The diagonal of a rectangular field is 60 meters more than the shorter side. If the longer side is 30 meters more than the shorter side, find the sides of the field.

Solution:

Let the shorter side of the rectangle be x m.

Then, longer side of the rectangle = $(x + 30)$ m

Diagonal of the rectangle = $\sqrt{x^2 + (x + 30)^2}$ (By Pythagoras theorem)

It is given that the diagonal of the rectangular field is 60 m more than the shorter side.

$$\sqrt{x^2 + (x + 30)^2} = x + 60$$

$$x^2 + (x + 30)^2 = (x + 60)^2 \quad \text{(By squaring on both the sides)}$$

$$x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$$

$$x^2 - 60x - 2700 = 0$$

$$x^2 - 90x + 30x - 2700 = 0$$

$$x(x - 90) + 30(x - 90)$$

$$(x - 90)(x + 30) = 0$$

$$x = 90 \text{ or } x = -30$$

Side of a rectangle cannot be negative. So, the length of the shorter side is 90 m.

Thus, the length of the longer side will be $(90 + 30)$ m = 120 m.

Question 7.

The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Solution:

Let the larger and smaller number be x and y respectively.

Given in the question,

$$x^2 - y^2 = 180$$

$$y^2 = 8x$$

$$x^2 - 8x = 180$$

$$x^2 - 8x - 180 = 0$$

$$x^2 - 18x + 10x - 180 = 0$$

$$x(x - 18) + 10(x - 18) = 0$$

$$(x - 18)(x + 10) = 0$$

$$x = 18, -10$$

If larger number, $x = -10$

Then smaller number, $y = \pm\sqrt{8x}$

$$= \pm\sqrt{8(-10)}$$

$$= \pm\sqrt{80}$$

Since we cannot have negative number in roots, $x = -10$ is not possible

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

Therefore, the larger number will be 18 only.

$$x = 18$$

$$y^2 = 8x = 8 \times 18$$

$$= 144$$

$$y = \pm\sqrt{144}$$

$$= \pm 12$$

Smaller number = ± 12

Therefore, the numbers are 18 and 12 or 18 and -12 .

Question 8.

A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution:

Let the speed to the train be x km/h.

$$\text{Total time taken to cover 360 km} = \frac{360}{x} \text{ hr} \quad \text{time} = \text{distance} / \text{speed}$$

Given in question,

$$(x + 5) \left(\frac{360}{x} - 1 \right) = 360 \quad (\text{Distance} = \text{Speed} \times \text{Time})$$

$$360 - x + \frac{1800}{x} - 5 = 360$$

$$\Rightarrow x^2 + 5x - 1800 = 0$$

$$\Rightarrow x^2 + 45x - 40x - 1800 = 0$$

$$\Rightarrow x(x + 45) - 40(x + 45) = 0$$

$$\Rightarrow (x + 45)(x - 40) = 0$$

$$\Rightarrow x = 40 \text{ or } x = -45$$

Speed cannot be negative. Thus, the speed of the train is 40 km/h.

Question 9.

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution:

Let the time taken by the smaller diameter tap to fill the tank be x hr.

$$\text{Time taken by the larger diameter tap} = (x - 10) \text{ hr}$$

$$\text{Volume of tank filled by smaller tap in 1 hour} = \frac{1}{x}$$

$$\text{Volume of tank filled by larger tap in 1 hour} = \frac{1}{x-10}$$

As per the question, the tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the taps together. Hence,

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

$$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\frac{x-10+x}{x(x-10)} = \frac{8}{75}$$

$$\frac{2x-10}{x(x-10)} = \frac{8}{75}$$

$$75(2x - 10) = 8x^2 - 80x$$

$$150x - 750 = 8x^2 - 80x$$

$$8x^2 - 230x + 750 = 0$$

$$8x^2 - 200x - 30x + 750 = 0$$

$$8x(x - 25) - 30(x - 25) = 0$$

$$(x - 25)(8x - 30) = 0$$

$$\text{i. e. , } x = 25 \text{ or } x = \frac{30}{8} = \frac{15}{4}$$

Time taken by smaller pipe cannot be $30/8$, as time taken by the larger pipe will become negative, which is not possible.

Thus, the time taken separately by the smaller diameter tap and the larger diameter tap will be 25 hours and $(25 - 10) = 15$ hours respectively.

Question 10.

An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Solution:

Let the average speed of passenger train be x km/h.

Average speed of express train = $(x + 11)$ km/h

According to the question, the time taken by the express train to cover 132 km is 1 hour less than a passenger train.

Therefore, time taken by passenger train – time taken by express train = 1 hour

$$\frac{132}{x} - \frac{132}{x+11} + 1 = 1 \quad \left(\text{Total time} = \frac{\text{Distance}}{\text{Average Speed}} \right)$$

$$132 \left[\frac{x+11-x}{x(x+11)} \right] = 1$$

$$\frac{132 \times 11}{x(x+11)} = 1$$

$$132 \times 11 = x(x + 11)$$

$$x^2 + 11x - 1452 = 0$$

$$x^2 + 44x - 33x - 1452 = 0$$

$$x(x + 44) - 33(9x + 44) = 0$$

$$(x + 44)(x - 33) = 0$$

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

$$x = -44, 33$$

Average speed of passenger train cannot be negative. Hence, the speed of the passenger train is 33km/h and thus, the speed of the express train will be $33 + 11 = 44$ km/h.

Question 11.

Sum of the areas of two squares is 468 m^2 . If the difference of their perimeters is 24 m, find the sides of the two squares.

Solution:

Let the sides of the two squares be x m and y m.

So, their perimeter will be $4x$ and $4y$ respectively and their areas will be x^2 and y^2 respectively.

According to the question,

$$4x - 4y = 24$$

$$x - y = 6$$

$$y = x + 6$$

$$\text{Also, } x^2 + y^2 = 468$$

$$(6 + y)^2 + y^2 = 468$$

$$36 + y^2 + 12y + y^2 = 468$$

$$2y^2 + 12y - 432 = 0$$

$$y^2 + 6y - 216 = 0$$

$$y^2 + 18y - 12y - 216 = 0$$

$$y(y + 18) - 12(y + 18) = 0$$

$$(y + 18)(y - 12) = 0$$

$$y = -18 \text{ or } 12.$$

Side of a square cannot be negative. Hence, $x = 12$ & $x = 12 + 6 = 18$

Therefore, the sides of the squares are 12 m and 18 m.