

Exercise No: 4.2

Question 1.

Find the roots of the following quadratic equations by factorization:

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Solution:

(i) The given quadratic expression can be written as follows:

$$x^2 - 3x - 10 = 0$$

$$x^2 - 5x + 2x - 10 = 0 \quad (\text{we factorize by method of splitting the middle term})$$

$$x(x - 5) + 2(x - 5) = 0$$

$$(x - 5)(x + 2) = 0$$

$$x - 5 = 0 \text{ or } x + 2 = 0$$

$$x = 5 \text{ or } x = -2$$

Hence, the roots of this quadratic equation are 5 and -2.

(ii) The given quadratic expression can be written as follows:

$$2x^2 + x - 6 = 0$$

$$2x^2 + 4x - 3x - 6 = 0 \quad (\text{we factorize by method of splitting the middle term})$$

$$2x(x + 2) - 3(x + 2) = 0$$

$$(x + 2)(2x - 3) = 0$$

$$x + 2 = 0 \text{ or } 2x - 3 = 0$$

$$x = -2 \text{ or } x = \frac{3}{2}$$

Hence, the roots of this quadratic equation are -2 and $\frac{3}{2}$.

(iii) The given quadratic expression can be written as follows:

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

$$\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0 \quad (\text{we factorize by method of splitting the middle term})$$

$$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$$

$$(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$$

$$x = \frac{-5}{\sqrt{2}} \text{ or } x = -\sqrt{2}$$

Hence, the roots of this quadratic equation are $-\frac{5}{\sqrt{2}}$ and $-\sqrt{2}$.

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

(iv) The given quadratic expression can be written as follows:

$$2x^2 - x + \frac{1}{8}$$

$$\frac{1}{8}(16x^2 - 8x + 1) = 0$$

$$(16x^2 - 4x - 4x + 1) = 0 \quad (\text{we factorize by method of splitting the middle term})$$

$$(4x(4x - 1) - 1(4x - 1)) = 0$$

$$(4x - 1)^2 = 0$$

$$x = \frac{1}{4} \text{ or } x = \frac{1}{4}$$

Hence, the roots of this quadratic equation are $\frac{1}{4}$ and $\frac{1}{4}$.

(v) The given quadratic expression can be written as follows:

$$100x^2 - 20x + 1$$

$$100x^2 - 10x - 10x + 1 = 0 \quad (\text{we factorize by method of splitting the middle term})$$

$$10x(10x - 1) - 1(10x - 1) = 0$$

$$(10x - 1)^2 = 0$$

$$x = \frac{1}{10} \text{ or } x = \frac{1}{10}$$

Hence, the roots of this quadratic equation are $\frac{1}{10}$ and $\frac{1}{10}$.

Question 2.

Solve the problems given below.

Represent the following situations mathematically:

- (i) **John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.**
- (ii) **A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day.**

Solution:

(i) Let the number of John's marbles be x .

The number of Jivanti's marbles = $45 - x$

If both lost 5 marbles each,

Then number of marbles left with John = $x - 5$

Then number of marbles left with Jivanti = $45 - x - 5$
 $= 40 - x$

Given that the product of their marbles is 124.

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

$$(x - 5)(40 - x) = 124$$

$$x^2 - 45x + 324 = 0$$

$$x^2 - 36x - 9x + 324 = 0$$

$$x(x - 36) - 9(x - 36) = 0$$

$$(x - 36)(x - 9) = 0$$

$$\text{Either } x - 36 = 0 \text{ or } x - 9 = 0$$

$$x = 36 \text{ or } x = 9$$

If the number of John's marbles = 36

Then, the number of Jivanti's marbles = $45 - 36 = 9$

If the number of John's marbles = 9

Then, the number of Jivanti's marbles = $45 - 9 = 36$.

(ii) Let the number of toys produced on that day be x .

The cost of production of each toy that day = ₹ $(55 - x)$

So, the total cost of production that day = $x(55 - x)$

As per the question, the total cost of production of the toys = ₹ 750

$$(55 - x)x = 750$$

$$x^2 - 55x + 750 = 0$$

$$x^2 - 25x - 30x + 750 = 0$$

$$x(x - 25) - 30(x - 25) = 0$$

$$(x - 25)(x - 30) = 0$$

$$\text{Either } x - 25 = 0 \text{ or } x - 30 = 0$$

$$x = 25 \text{ or } x = 30$$

Thus, the number of toys produced that day will be either 25 or 30.

Question 3.

Find two numbers whose sum is 27 and product is 182.

Solution:

Let the first number be x .

Then the second number is $27 - x$.

According to the question, the product of these numbers is 182.

Therefore, $x(27 - x) = 182$

$$x^2 - 27x + 182 = 0$$

$$x^2 - 13x - 14x + 182 = 0$$

$$x(x - 13) - 14(x - 13) = 0$$

$$(x - 13)(x - 14) = 0$$

$$\text{Either } x - 13 = 0 \text{ or } x - 14 = 0$$

$$x = 13 \text{ or } x = 14$$

If first number = 13,

Then Second number = $27 - 13 = 14$

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

If first number = 14,

Then Second number = $27 - 14 = 13$

Hence, the numbers are 13 and 14.

Question 4.

Find two consecutive positive integers, sum of whose squares is 365.

Solution:

Let the consecutive positive integers be x and $x + 1$.

As per the question,

$$x^2 + (x + 1)^2 = 365$$

$$x^2 + x^2 + 1 + 2x = 365$$

$$2x^2 + 2x - 364 = 0$$

$$x^2 + x - 182 = 0$$

$$x^2 + 14x - 13x - 182 = 0$$

$$x(x + 14) - 13(x + 14) = 0$$

$$(x + 14)(x - 13) = 0$$

Either $x + 14 = 0$ or $x - 13 = 0$,

$$x = -14 \text{ or } x = 13$$

Since, given that integers are positive, x can only be 13.

$$x + 1 = 13 + 1$$

$$= 14$$

Hence, the two consecutive positive integers are 13 and 14.

Question 5.

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution:

As per the question, hypotenuse is 13 cm

Let the base of the right triangle be x cm.

Its altitude = $(x - 7)$ cm

Using Pythagoras theorem,

$$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$$

$$x^2 + (x - 7)^2 = 13^2$$

$$x^2 + x^2 + 49 - 14x = 169$$

$$2x^2 - 14x - 120 = 0$$

$$x^2 - 7x - 60 = 0$$

$$x^2 - 12x + 5x - 60 = 0$$

$$x(x - 12) + 5(x - 12) = 0$$

$$(x - 12)(x + 5) = 0$$

Either $x - 12 = 0$ or $x + 5 = 0$,

NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations

$$x = 12 \text{ or } x = -5$$

Since, sides of a triangle are positive, x can only take 12.

Hence, the base of the right triangle is 12 cm and the altitude of this triangle is

$$(12 - 7) \text{ cm} = 5 \text{ cm.}$$

Question 6.

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹. 90, find the number of articles produced and the cost of each article.

Solution:

Let the number of articles produced on that day be x .

The cost of production of each article = ₹ $(2x + 3)$

According to the question, the total cost of production is ₹ 90.

$$x(2x + 3) = 90$$

$$2x^2 + 3x - 90 = 0$$

$$2x^2 + 15x - 12x - 90 = 0$$

$$x(2x + 15) - 6(2x + 15) = 0$$

$$(2x + 15)(x - 6) = 0$$

Either $2x + 15 = 0$ or $x - 6 = 0$,

$$x = \frac{-15}{2} \text{ or } x = 6$$

It's clear that number of articles produced can only be a positive integer, so, x can only be 6.

Therefore, number of articles produced on that day = 6

Cost of each article = $(2 \times 6) + 3 = ₹ 15$