Exercise 4.1

Question 1:

Check whether the following are quadratic equations:

(i) $(x + 1)^2 = 2(x - 3)$

(ii) $x^2 - 2x = (-2)(3 - x)$

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iv) $(x - 3)(2x + 1) = x(x + 5)$

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

(vi) $x^2 + 3x + 1 = (x - 2)^2$

(vii) $(x + 2)^2 = 2x(x^2 - 1)$

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Solution:

(i) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

Given equation:

$(x + 1)^2 = 2(x - 3)$

Using the formula $(a + b)^2 = a^2 + 2ab + b^2$

$x^2 + 2x + 1 = 2x - 6$

$x^2 + 7 = 0$

Here, $a = 1$, $b = 0$ and $c = 7$.

Thus, the given equation is a quadratic equation as $a \neq 0$.

(ii) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

Given equation:

$x^2 - 2x = (-2)(3 - x)$

$x^2 - 2x = -6 + 2x$

$x^2 - 4x + 6 = 0$

Here, $a = 1$, $b = -4$ and $c = 6$.

Thus, the given equation is a quadratic equation as $a \neq 0$.

(iii) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$

Given equation:

$(x - 2)(x + 1) = (x - 1)(x + 3)$

$x^2 - x - 2 = x^2 + 2x - 3$

$3x - 1 = 0$

Here $a = 0$.

Thus, the given equation is not a quadratic equation.

(iv) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

Given equation:

$(x - 3)(2x + 1) = (x + 5)$

$2x^2 - 5x - 3 = x^2 + 5x$

$x^2 - 10x - 3 = 0$

Here, $a = 1$, $b = -10$ and $c = -3$.

Thus, the given equation is a quadratic equation as $a \neq 0$.

(v) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

Given equation:

$(2x - 1)(x - 3) = (x + 5)(x - 1)$

$2x^2 - 7x + 3 = x^2 + 4x - 5$

$x^2 - 11x + 8 = 0$

Here, $a = 1$, $b = -11$ and $c = 8$.

Thus, the given equation is a quadratic equation as $a \neq 0$.

(vi) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

Given equation: $x^2 + 3x + 1 = (x - 2)^2$

Using the formula $(a - b)^2 = a^2 - 2ab + b^2$

$x^2 + 3x + 1 = x^2 - 4x + 4$

$7x - 3 = 0$

But, here $a = 0$.

Thus, the given equation is not a quadratic equation.

(vii) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.

Given equation: $(x + 2)^3 = 2(x^2 - 1)$

Using the formula $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x$

$x^3 - 14x - 6x^2 - 8 = 0$

This equation is not of the form $ax^2 + bx + c = 0$

Thus, the given equation is not a quadratic equation.

(viii) We know that any equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b, c are real numbers and $a \neq 0$.
Given equation: $x^3 - 4x^2 - x + 1 = (x - 2)^3$
Using the formula $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$
$2x^2 - 13x + 9 = 0$
Here, a = 2, b = −13 and c = 9.
Thus, the given equation is a quadratic equation as $a \neq 0$.

Question 2.

Represent the following situations in the form of quadratic equations:

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in meters) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Solution:

(i)
Let the breadth of the plot be x m.
Therefore, the length of the plot is (2x + 1) m. (Since, given that length is one more than twice its breadth)

Area of a rectangle = length × breadth
Given: area of rectangle = 528 m^2
$528 = x(2x + 1)$
$2x^2 + x - 528 = 0$ ..... (i), which is of the form $ax^2 + bx + c = 0$
Here $a = 2 (\neq 0)$, b = 1 and c = −528
Thus, length and breadth of the plot satisfies the quadratic equation $2x^2 + x - 528 = 0$.

(ii)
Let the consecutive positive integers be x and x + 1.
Given that their product is 306.
$x(x + 1) = 306$

$x^2 + x - 306 = 0$ ……. (i), which is of the form $ax^2 + bx + c = 0$

Here a = 1(≠ 0), b = 1 and c = −306

Thus, two integers x and x +1 satisfies the quadratic equation $x^2 + x - 306 = 0$.

(iii)

Let Rohan's age be x,

His mother's age = x + 26

After 3 years

Rohan's age will be = x + 3

Mother's age will be = x + 26 + 3

= x + 29

Given that the product of their ages after 3 years is 360.

(x + 3)(x + 29) = 360

On simplification, we get

$x^2 + 32x - 273 = 0$ …… (i) Which is of the form $ax^2 + bx + c = 0$

Here a = 1(≠ 0), b = 32 and c = −273

Thus, the age of rohan satisfies the quadratic equation $x^2 + 32x - 273 = 0$.

(iv)

Let the speed of train be x km/h.

Total time taken to travel 480 km = $\frac{480}{x}$ hrs

Given: speed became 8 km/h less So, the speed of train = (x − 8)km/h

The train will take 3 more hours to cover the same distance. Therefore, time take to travel 480 km = ($\frac{480}{x}$ + 3) hrs

Speed × Time = Distance

$(x - 8)\left(\frac{480}{x} + 3\right) = 480$

$480 + 3x - \frac{3840}{x} - 24 = 480$

$3x - \frac{3840}{x} = 24$

$3x^2 - 24x - 3840 = 0$

$x^2 - 8x - 1280 = 0$ …….. (i) Which is of the form $ax^2 + bx + c = 0$

Here a = 1(≠ 0), b = −8 and c = −1280

Thus, the speed of the train satisfies the quadratic equation $x^2 - 8x - 1280 = 0$.