Polynomials

Polynomials can be linear (x), quadratic (x^2), cubic (x^3) and so on, depending on the highest power of the variable.

The number of zeroes of a polynomial is equal to the degree of the polynomial, and there is a well-defined mathematical relationship between the zeroes and the coefficients.

What are zeroes of a Polynomial?

- A zero is a value for which a polynomial is equal to zero.
  - When you set a polynomial equal to zero, then you have a polynomial equation where the equations roots are same as the polynomial's zeroes.
- A root is a value for which a polynomial equation is true.
- Example: The polynomial x - 5 has one zero, that is x = 5. And the polynomial equation x - 5 = 0 has one root, that is, x = 5.

Linear Polynomial:-

Any equation of the first degree is known as a linear equation. It is an equation of the form ax + b = 0, where and b are constants and x is a variable.

The general form of a linear polynomial is p(x) = ax + b, its zero is

$-b/a = -$ (constant term)/ (coefficient of x)

Quadratic Polynomial:-

General form of a quadratic polynomial is $ax^2 + bx + c$ where $a \neq 0$. There are two zeroes, say α and β of a quadratic polynomial, where

Sum of the roots = $\alpha + \beta = -b / a$

Product of the toots = $\alpha\beta = c / a$

Cubic Polynomial:-

If α, β ,γ are the zeros of a cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$, then

$A + \beta + \gamma = -b / a$.

$\alpha\beta + \beta\gamma + \gamma\alpha = c/a.$

$\alpha\beta\gamma = -d/a.$

Division Algorithm:-

Let f(x), g(x), q(x) and r(x) are polynomials then the division algorithm for polynomials states that "If f(x) and g(x) are two polynomials such that degree of f(x) is greater than degree of g(x) where g(x) ≠ 0, then there exists unique polynomials q(x) and r(x) such that f(x) = g(x).q(x) + r(x) where r(x) = 0 or degree of r(x) < degree of g(x).

OR Dividend = divisor x quotient + remainder

$$\begin{array}{r|l} & 3x^2 + 4x + 5 \\ \hline x+4 & 3x^3 + 16x^2 + 21x + 20 \\ & 3x^3 + 12x^2 \\ & - \quad - \\ \hline & 4x^2 + 21x + 20 \\ & 4x^2 + 16x \\ & - \quad - \\ \hline & 5x + 20 \\ & 5x + 20 \\ & - \quad - \\ \hline & 0 \end{array}$$

First term of q(x) $= \dfrac{3x^3}{x} = 3x^2$

Second term of q(x) $= \dfrac{4x^2}{x} = 4x$

Third term of q(x) $= \dfrac{5x}{x} = 5$