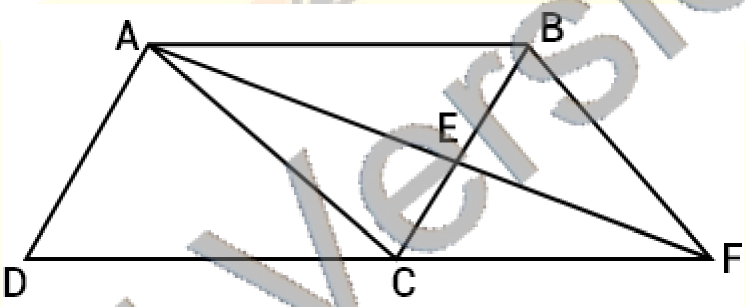
Exercise No. 9.4

Long Answer Questions:

**1. A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that
ar (ADF) = ar (ABFC)**

Solution:
Given in the question, A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F.
Prove that ar (ADF) = ar (ABFC)
Proof: ABCD is a parallelogram and AC divides it into two triangle of equal area.



$ar(\Delta ADC) = ar(\Delta ABC)$
So, DC||AB and CF||AB

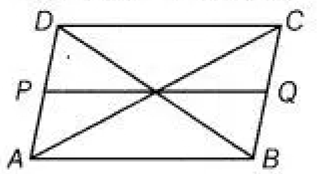
As we know that triangle on the same base and between the same parallels are equal in area. So,
$ar(\Delta ACF) = ar(\Delta BCF) \quad \ldots(II)$

Adding equation (I) and (II), get:
$ar(\Delta ADC) + ar(ACF) = ar(\Delta ABC) + ar(\Delta BCF)$
$ar(\Delta ADF) = ar(ABFC)$
Hence, proved.

2. The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. show that PQ divides the parallelogram into two parts of equal area.

Solution:
Given: ABCD is a parallelogram and diagonal interact at O, and draw a line PQ which intersects AD and BC.

To prove that PQ divides the parallelogram ABCD into two parts of equal area that $ar(ABQP) = ar(CDPQ)$.



Proof: AC is a diagonal of the parallelogram ABCD.

$ar\left(\Delta \frac{1}{2} ACD\right) = \frac{1}{2} ar(ABCD)$ …(I)

In triangle AOP and triangle COQ,

AO = CO [Diagonals of a parallelogram bisect each other]

$\angle AOP = \angle COQ$ [Vertical opposite angles]

$\angle OAP = \angle OCQ$ [Alternate angles, AB||CD]

$\Delta AOP = \Delta COQ$ [By ASA congruent rule]

Since, $ar(\Delta AOP) = ar(\Delta COQ)$ [Congruent area axiom] …(II)

Now, adding $ar(AOQD)$ to both sides of (II), get:

$ar(\Delta ACD) = \frac{1}{2} ar(ABCD)$ [From equation (I)]

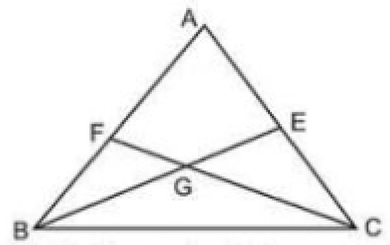
Hence, $ar(APQD) = \frac{1}{2} ar(ABCD)$.

3. The medians BE and CF of a triangle ABC intersect at G. Prove that the area of ΔGBC = area of the quidrilateral AFGE

Solution:

Given: The medians BE and CF of a triangle ABC intersect at G

To prove that $ar(\Delta GBC) = ar(AFGE)$.



Proof: As median CF divides a triangle into two triangle of equal area. So,

$ar(\Delta BCF) = ar(\Delta ACF)$

$ar(\Delta GBF) + ar(\Delta GBC) = ar(AFGE) + ar(\Delta GCE)$ …(I)

Now, median BE divides a triangle into two triangle of equal area. So,

$ar(\Delta GBF) + ar(AFGE) = ar(\Delta GCE) + ar(\Delta GBC)$ …(II)

Now, subtracting (II) from (I), get:

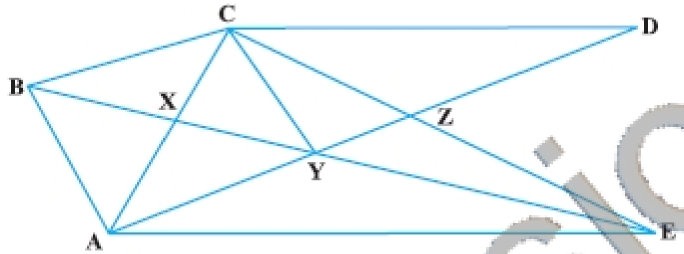
$$ar(\Delta GBC) - ar(AFGE) = ar(\Delta AFGE) - ar(\Delta GBC)$$

$$ar(\Delta GBC) + ar(\Delta GBC) = ar(\Delta AFGE) + ar(\Delta AFGE)$$

$$2ar(\Delta GBC) = 2ar(AFGE)$$

Hence, $ar(\Delta GBC) = ar(AFGE)$.

4. In Fig., CD || AE and CY || BA. Prove that ar (CBX) = ar (AXY)



Solution:

Given: CD || AE and CY || BA

To prove that ar (CBX) = ar (AXY).

Proof: As we know that triangle on the same base and between the same parallels are equal in area. So,

$$ar(\Delta ABC) = ar(\Delta ABY)$$

$$ar(\Delta CBX) + ar(\Delta ABX) = ar(\Delta ABX) + ar(\Delta AXY)$$

Hence, $ar(\Delta CBX) = ar(\Delta AXY)$.

5. ABCD is a trapezium in which AB || DC, DC = 30 cm and AB = 50 cm. If X and Y are, respectively the mid-points of AD and BC, prove that

$$\mathrm{ar}(\mathrm{DCYX}) = \frac{7}{9}\mathrm{ar}(\mathrm{XYBA})$$

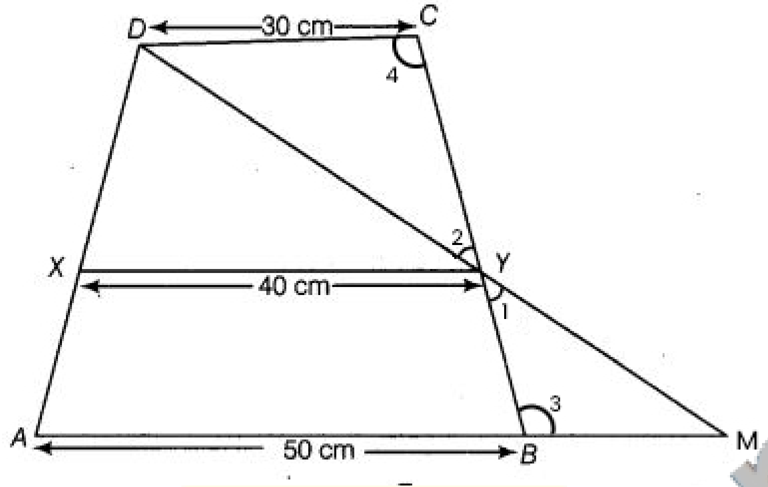
Solution:

To prove that $\mathrm{ar}(\mathrm{DCYX}) = \frac{7}{9}\mathrm{ar}(\mathrm{XYBA})$

Proof: In triangle MBY and triangle DCY,

$\angle 1 = \angle 2$ [Vertically opposite angles]

$\angle 3 = \angle 4$ [AB||DC and alternate angles are equal]



BY = CY [Y is the mid-point of BC]
$\Delta MBY \cong \Delta DCY$ [By ASA congruent angle]
So, MB = DC = 30 cm [CPCT]
Now, AM = AB + BM
=50cm+30cm
=80cm

In triangle ADM,
$$XY = \frac{1}{2}AM = \frac{1}{2}\times 80\text{cm} = 40\text{cm}$$

Now, AB||XY||DC and X and Y are the mid-points of AD and BC, So, height of trapezium DCXY and XYBA are equal and assume the equal height be h cm.

$$\frac{ar(DCYX)}{ar(XYBA)} = \frac{\frac{1}{2}\times(30+40)\times h}{\frac{1}{2}\times(30+50)\times h} = \frac{70}{90} = \frac{7}{9}$$

Hence, $ar(DCYX) = \frac{7}{9}ar(XYBA)$.

6. In ΔABC, if L and M are the points on AB and AC, respectively such that LM || BC. Prove that
ar (LOB) = ar (MOC)

Solution:
Given: in triangle ABC and L and M are the points on AB and AC, respectively such that LM || BC.
Prove that ar (LOB) = ar (MOC)

Proof: As we know that triangle on the same base and between the same parallels are equal in area.
$ar(\Delta LBM) = ar(\Delta LCM)$

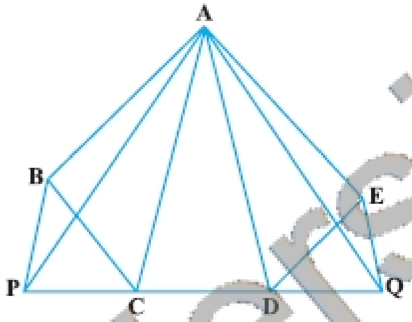
[Triangle LBM and triangle LCM are on the same base LM and between the same parallels LM and BC]

$$ar(\Delta LBM) = ar(\Delta LCM)$$

$$ar(\Delta LOM) + ar(\Delta LOB) = ar(\Delta LOM) + ar(\Delta MOC)$$

Hence, $ar(\Delta LOB) = ar(\Delta MOC)$.

7. In Fig., ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that
ar (ABCDE) = ar (APQ)



Solution:
Given: ABCDE is any pentagon and BP || AC meets DC produced at P and EQ || AD meets CD produced at Q.
Prove that ar (ABCDE) = ar (APQ)

Proof: As we know that triangle on the same base and between the same parallels are equal in area.

$ar(\Delta ABC) = ar(\Delta APC)$ ...(I)

$ar(\Delta ADE) = ar(\Delta ADQ)$ ...(II)

Now, adding equation (I) and (II), get:

$$ar(\Delta ABC) + ar(\Delta ADE) = ar(\Delta APC) + ar(\Delta ADQ)$$

Now, adding $ar(\Delta ACD)$ to both side, get:

$$ar(\Delta ABC) + ar(\Delta ADE) + ar(ACD) = ar(\Delta APC) + ar(\Delta ADQ) + ar(ACD)$$

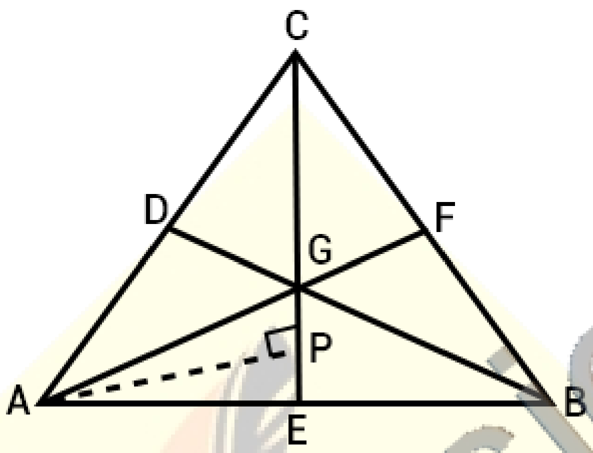
Hence, $ar(ABCDE) = ar(\Delta APQ)$.

8. If the medians of a ΔABC intersect at G, show that
ar (AGB) = ar (AGC) = ar (BGC)= $\frac{1}{3}$ ar (ABC)

Solution:

Given: The median of a triangle ABC intersect at G.

To prove that ar (AGB) = ar (AGC) = ar (BGC) = $\frac{1}{3}$ ar (ABC)



Construction: Draw BP ⊥ EG

Proof: $AG = \frac{2}{3} AE$ [Centroid divides the median in the ration 2:1]

Now, $ar(\Delta AGB) = \frac{1}{2} \times AG \times BP$

[Median divides a triangle into two triangles equal in area]

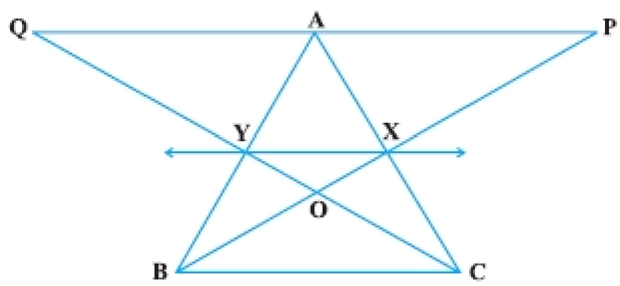
$= \frac{1}{3} ar(\Delta ABC)$

Again, $ar(\Delta AGC) = ar(\Delta BGC) = \frac{1}{3} ar(\Delta ABC)$

So, $ar(\Delta AGB) = ar(\Delta AGC) = ar(\Delta BGC) = \frac{1}{3} ar(\Delta ABC)$

Hence, proved.

9. In Fig., X and Y are the mid-points of AC and AB respectively, QP || BC and CYQ and BXP are straight lines. Prove that ar (ABP) = ar (ACQ).



Solution:

Given: In triangle ABC, X and Y are the mid-points of AB and AC.

To prove that ar (ABP) = ar (ACQ).

Proof: Since, XY||BY [BY mid-point theorem]

As we know that triangle on the same base and between the same parallels lines are equal in area. So,

$ar(\Delta BYC) = ar(\Delta BXC) \quad \ldots(I)$

Now, subtracting $ar(\Delta BOC)$ from both sides in the above, get:

$ar(\Delta BYC) - ar(\Delta BOC) = ar(\Delta BXC) - ar(\Delta BOC)$

$ar(\Delta BOY) = ar(\Delta COX) \ldots(II)$

Now, adding $ar(\Delta XOY)$ to both side in equation (II), get:

$ar(\Delta BOY) + ar(\Delta XOY) = ar(\Delta COX) + ar(\Delta XOY) \quad \ldots(III)$

Again, quadrilaterals XYAP and YXAQ are on the same base XY and between the same parallels XY and PQ. So,

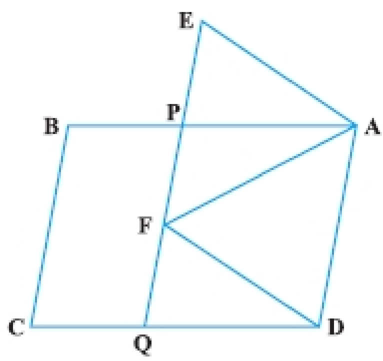
$ar(XYAP) = ar(XYQA) \quad \ldots(IV)$

Now, adding equation (III) and (IV), get:

$ar(\Delta BXY) + ar(XYAP) = ar(\Delta CXY) + ar(XYAQ)$

Hence, $ar(\Delta ABP) = ar(\Delta ACQ)$.

10. In Fig., ABCD and AEFD are two parallelograms. Prove that ar (PEA) = ar (QFD) [Hint: Join PD].

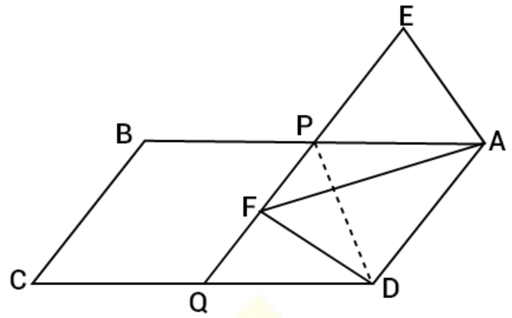


Solution:

Given: ABCD and AEFD are two parallelogram.

To prove that $ar(\Delta PEA) = ar(\Delta QFD)$

Construction: join PD.



Proof: In triangle PEA and triangle QFD,

$\angle APE = \angle DQF$ [Corresponding angles are equal as AB||CD]

$\angle AEP = \angle DEQ$ [Corresponding angles are equal as AE||DF]

AE = DF [Opposite sides of a parallelogram are equal]

So, $\Delta PEA \cong \Delta QFD$ [By AAS congruent rule]

Hence, $ar(\Delta PEA) = ar(\Delta QED)$.