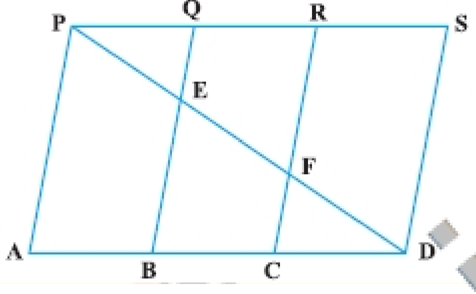
Exercise No. 9.3

Short Answer Questions:

1. In Fig., PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR = RS and PA || QB || RC. Prove that ar (PQE) = ar (CFD).



Solution:

Given: PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR = RS and PA || QB || RC.

To prove that ar (PQE) = ar (CFD).

Proof: PS = AD [opposite angle of a parallelogram]

$\frac{1}{3}PS = \frac{1}{3}AD$

PQ = CD …(I)

Similarly, PS||AD and QB cut them. So,

$\angle PQE = \angle CBE$ [Alternate angles]…(II)

Again, QB||RC and AD cut them,

$\angle QBD = \angle RCD$ [Corresponding angle] …(III)

So, $\angle PQE = \angle FCD$ …(IV) [From (II) and (III), $\angle CBE$ and $\angle QBD$ are same and $\angle RCD$ and $\angle FCD$ are same]

Now, in triangle PQE and triangle CFD,

$\angle PQE = \angle CDF$ [Alternate angle]

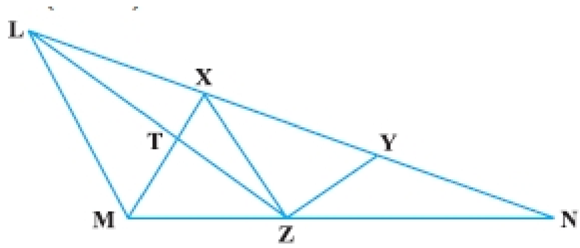
PQ = CD [From equation (I)]

$\angle QPE = \angle FCD$ [From equation (IV)]

$\Delta PQE \cong \Delta CFD$ [By ASA congruence rule]

Hence, $ar(\Delta PQE) = ar(\Delta CFD)$. [Congruence triangle are equal in area]

2. X and Y are points on the side LN of the triangle LMN such that LX = XY = YN. Through X, a line is drawn parallel to LM to meet MN at Z (See Fig.). Prove that
ar (LZY) = ar (MZYX)



Solution:

Prove that ar (LZY) = ar (MZYX)

Proof: As ΔLXZ and ΔXMZ are on the same base and between the same parallels LM and XZ.

$ar(\Delta LXZ) = ar(\Delta XMZ)$

Now, adding $ar(\Delta XYZ)$ to both sides of (I), get:

$ar(\Delta LXZ) + ar(\Delta XYZ) = ar(\Delta XMZ) + ar(\Delta XYZ)$

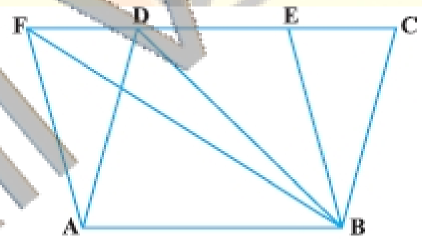
$ar(\Delta LZY) = ar(MZYX)$

3. The area of the parallelogram ABCD is 90 cm^2 (see Fig.). Find

(i) ar (ABEF)

(ii) ar (ABD)

(iii) ar (BEF)



Solution:

(i) As we know that parallelogram on the same base and between the same parallels are equal in area.

$ar(ABEF) = ar(ABCD)$

Hence, $ar(ABEF) = ar(ABCD) = 90\text{cm}^2$.

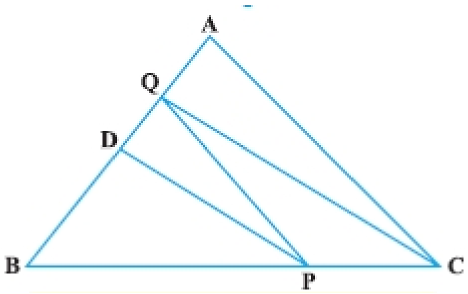
(ii) $ar(\Delta ABD) = \frac{1}{2} ar(ABCD)$ [A diagonal of a parallelogram divides the parallelogram in two triangle of equal area]

$= \frac{1}{2} \times 90\text{cm}^2 = 45\text{cm}^2$

(iii) $ar(\Delta BEF) = \frac{1}{2} ar(ABEF) = \frac{1}{2} \times 90\text{cm}^2 = 45\text{cm}^2$

4. In ΔABC, D is the mid-point of AB and P is any point on BC. If CQ || PD meets AB in Q
(Fig.), then prove that

$ar(BPQ) = \frac{1}{2} ar(ABC)$

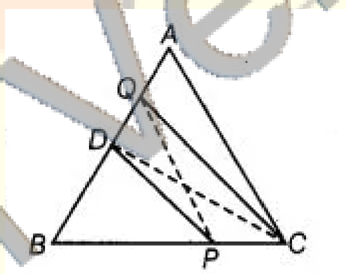


Solution:

Given in triangle ABC, D is the mid-point of AB and P is any point on BC.
CQ||PD means AB in Q.

To prove that $ar(\Delta BPQ) = \frac{1}{2} ar(\Delta ABC)$

Construction: Join PQ and CD.



Proof:
As we know that median of a triangle divides it into two triangles of equal area. So,

$ar(\Delta BCD) = \frac{1}{2} ar(\Delta ABC)$ ...(I)

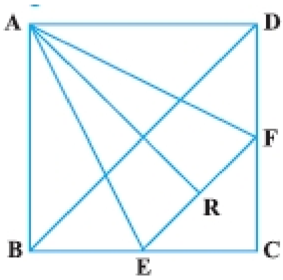
Also, we know that triangles on the same base and between the same parallels are equal in area. So,

$ar(\Delta DPQ) = ar(\Delta DPC)$ [Triangle DPQ and DPC are on the same base DP and between the same parallels DP and CQ]

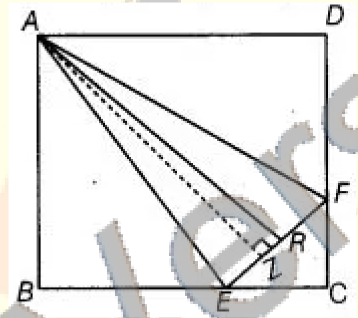
$ar(\Delta DPQ) + ar(\Delta DPB) = ar(\Delta DPC) + ar(DPB)$

Hence, $ar(\Delta BPQ) = ar(\Delta BCD) = \frac{1}{2} ar(\Delta ABC)$.

5. ABCD is a square. E and F are respectively the midpoints of BC and CD. If R is the mid-point of EF (Fig.), prove that
ar (AER) = ar (AFR)

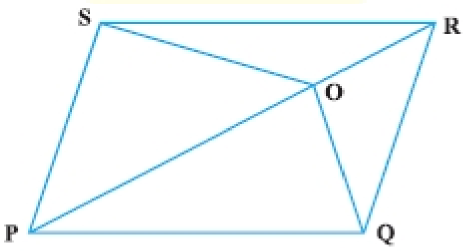


Solution:
Given: ABCD is a square. E and F are respectively the midpoints of BC and CD. Also, R is the mid-point of EF.
To prove that $ar(\Delta AER) = ar(\Delta AFR)$
Construction: Draw AN $\perp$ EF
Proof:



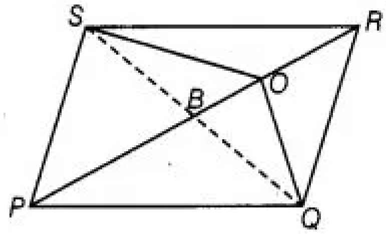
$$ar(\Delta AER) = \frac{1}{2} \times \text{Base} \times \text{Height}$$
$$= \frac{1}{2} \times ER \times AN$$
$$= \frac{1}{2} \times FR \times AN \quad \text{[R is the mid-point of EF so ER = FR]}$$
$$= ar(\Delta AFR)$$

**6. O is any point on the diagonal PR of a parallelogram PQRS (Fig.). Prove that
ar (PSO) = ar (PQO).**



Solution:
Given: O is any point on the diagonal PR of a parallelogram PQRS.
To prove that ar (PSO) = ar (PQO).

Construction: Join SQ which intersects PR at B.



Proof: B is the mid-point of SQ because diagonal of a parallelogram bisect each other.
See the above figure, PB is a median of ΔQPS and as we know that a median of a triangle divides it into two triangle of equal area.

$ar(\Delta BPQ) = ar(\Delta BPS)$ ...(I)

Similarly, OB is the median of ΔOSQ.

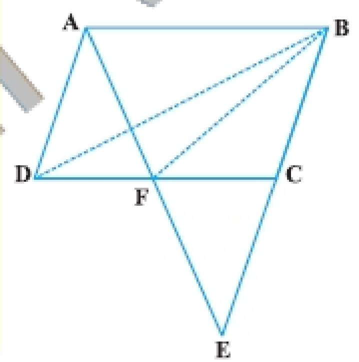
$ar(\Delta OBQ) = ar(OBS)$ ...(II)

Now, adding equation (I) and (II), get:

$ar(\Delta BPQ) + ar(\Delta OBQ) = ar(\Delta BPS) + ar(\Delta OBS)$

$ar(\Delta PQO) = ar(\Delta PSO)$

7. ABCD is a parallelogram in which BC is produced to E such that CE = BC (Fig.). AE intersects CD at F.
If $\text{ar}(\text{DFB}) = 3\text{ cm}^2$, find the area of the parallelogram ABCD.



Solution:

Given: ABCD is a parallelogram in which BC is produced to E such that CE = BC. C is the mid-point BE and $ar(\Delta DFB) = 3\text{cm}^2$.

In triangle ADF and triangle EFC,

$\angle DAF = \angle CEF$ [Alternate interior angle]

AD = CE [AD = BC = CE]

$\angle ADF = \angle FCE$ [Alternate interior angle]

So, $\Delta ADF \cong \Delta ECF$ [By SAS rule of congruence]

Now, $\Delta ADF \cong \Delta ECF$ [By SAS rule of congruence]

$DF = CF$ [CPCT]

As BF is the median of triangle BCD.

$ar(\Delta BDF) = \frac{1}{2} ar(BCD)$ …(I) [Median divides a triangle into two triangle of equal area]

As we know that a triangle and parallelogram are on the same base and between the same parallels then the area of the triangles is equal to half the area of the parallelogram.

$ar(\Delta BCD) = \frac{1}{2} ar(ABCD)$ …(II)

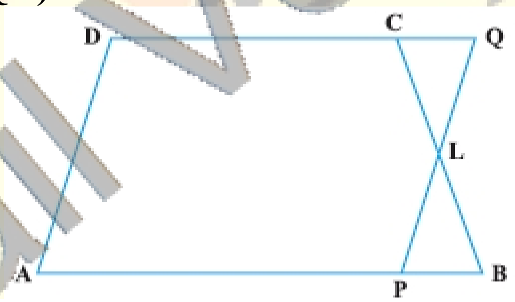
$ar(\Delta BDF) = \frac{1}{2}\left\{\frac{1}{2} ar(ABCD)\right\}$ [By equation (I)]

$3\text{cm}^2 = \frac{1}{4} ar(ABCD)$

$ar(ABCD) = 12\text{cm}^2$

Hence, the area of the parallelogram is $12\,\text{cm}^2$.

8. In trapezium ABCD, AB || DC and L is the mid-point of BC. Through L, a line PQ || AD has been drawn which meets AB in P and DC produced in Q (Fig.). Prove that
ar (ABCD) = ar (APQD)



Solution:
To prove that ar (ABCD) = ar (APQD)
Proof: AS AB||DC and AB||DQ

In triangle CLQ and triangle BLP,
$\angle QCL = \angle LBP$ [Alternate angles]
CL = LP [L is the mid-point og BC]
$\angle CLQ = \angle BLP$ [Vertical opposite angles]
$\Delta CLQ \cong \Delta BLP$ [By ASA congruence rule]
So, $ar(\Delta CLQ) = ar(\Delta BLP)$ …(I) [Congruent triangles are equal in area]

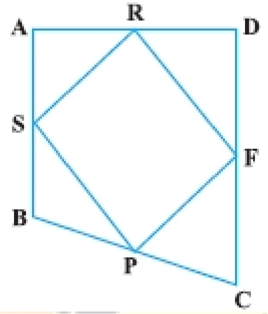
Now, adding $ar(APLCD)$ both side in above equation, get:

$ar(\Delta CLQ) + ar(APLCD) = ar(\Delta BLP) + ar(APLCD)$

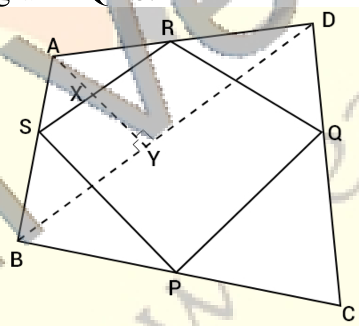
$ar(\Delta APQD) = ar(ABCD)$

Hence, proved.

9. If the mid-points of the sides of a quadrilateral are joined in order, prove that the area of the parallelogram so formed will be half of the area of the given quadrilateral (Fig.).
[Hint: Join BD and draw perpendicular from A on BD.]



Solution:
According to the question, a quadrilateral ABCD in which the mid-point of the sides of it are joined in order of form parallelogram PQRS.



To Prove that $ar(PQRS) = \frac{1}{2} ar(ABCD)$

Construction: Join BD and draw perpendicular from A and BD which interest SR and BD at X and Y respectively.
Proof: In triangle ABD, S and R are the mid-points of sides AB and AD respectively. So, $SR \parallel BD$

And: $ASX \parallel BY$

See the figure, x is the mid-point of AY. So,
AX = XY

And $SR = \frac{1}{2} BD$ …(II)[mid-point theorem]

Now, $ar(\Delta ABD) = \frac{1}{2} \times BD \times AY$

$ar(\Delta ASR) = \frac{1}{2} \times SR \times AX$

$ar(\Delta ASR) = \frac{1}{2} \times \left(\frac{1}{2} BD\right) \times \left(\frac{1}{2} AY\right)$ [Using equation (I) and (II)]

$ar(\Delta ASR) = \frac{1}{4} \times \left(\frac{1}{2} BD \times AY\right)$

$ar(\Delta ASR) = \frac{1}{4} \times (\Delta ABD) \quad \ldots(\text{III})$

Again, $ar(\Delta CPQ) = \frac{1}{4} ar(\Delta CBD) \quad \ldots(\text{IV})$

$ar(\Delta BPS) = \frac{1}{4} ar(\Delta BAC) \quad \ldots(\text{V})$

$ar(\Delta DRQ) = \frac{1}{4} ar(DAC) \quad \ldots(\text{VI})$

Now, adding equation (III), (IV), (V) and (VI), get:

$ar(\Delta ASR) + ar(\Delta CPQ) + ar(BPS) + ar(\Delta DRQ)$

$= \frac{1}{4} ar(\Delta ABD) + \frac{1}{4} ar(\Delta CBD) + \frac{1}{4} ar(\Delta ABC) + \frac{1}{4} ar(\Delta DAC)$

$= \frac{1}{4}\left[ar(\Delta ABD) + ar(\Delta CBD) + ar(\Delta ABC) + ar(\Delta DAC)\right]$

$= \frac{1}{4}\left[ar(ABCD) + ar(ABCD)\right]$

$= \frac{1}{4} \times 2ar(ABCD)$

$= \frac{1}{2} ar(ABCD)$

So, $ar(\Delta ASR) + ar(\Delta CPQ) + ar(\Delta BPS) + ar(\Delta DRQ) = \frac{1}{2} ar(ABCD)$

$ar(ABCD) - ar(PQRS) = \frac{1}{2} ar(ABCD)$

Now, $ar(PQRS) = ar(ABCD) - \frac{1}{2} ar(ABCD)$

$ar(PQRS) = \frac{1}{2} ar(ABCD)$

Hence, proved.