Exercise 5.3

Short Answer Questions:

Question:

1. Match the APs given in column A with suitable common differences given in column B.

Column A	**Column B**
(A_1) $2, -2, -6, -10, \ldots$	(B_1) $\frac{2}{3}$
(A_2) $a = -18, n = 10, a_n = 0$	(B_2) -5
(A_3) $a = 0, a_{10} = 6$	(B_3) 4
(A_4) $a_2 = 13, a_4 = 3$	(B_4) -4
	(B_5) 2
	(B_6) $\frac{1}{2}$
	(B_7) 5

Solution:

(A_1)

AP is 2, – 2, – 6, – 10…

So,

$d = a_2 - a_1$

$= -2 - 2$

$= -4$

$= (B_3)$

(A_2)

First term, a = – 18

No of terms, n = 10

Last term, an = 0

We have,

$a_n = a + (n - 1)d$

$0 = -18 + (10 - 1)d$
$18 = 9d$
$d = 2 = (B_5)$

(A_3)
First term, $a = 0$
Tenth term, $a_{10} = 6$

We have,
$a_n = a + (n - 1)d$
$a_{10} = a + 9d$
$6 = 0 + 9d$
$d = \frac{2}{3} = (B_6)$

(A_4)
Taking the first term be a and common difference be d

We have,
$a_2 = 13$
$a_4 = 3$
$a_2 - a_4 = 10$
$a + d - (a + 3d) = 10$

$d - 3d = 10$
$-2d = 10$
$d = -5$
$= (B_1)$

2. Verify that each of the following is an AP, and then write its next three terms.

i. $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

ii. $5, \frac{14}{3}, \frac{13}{3}, 4, \ldots$

iii. $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$

iv. $a+b, (a+1)+b, (a+1)+(b+1), \ldots$

v. $a, 2a+1, 3a+2, 4a+3, \ldots$

Solution:
(i)
$0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

$a_1 = 0$

$a_2 = \frac{1}{4}$

$a_3 = \frac{1}{2}$

$a_4 = \frac{3}{4}$

$a_2 - a_1 = \frac{1}{4} - 0 = \frac{1}{4}$

$a_3 - a_2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$a_4 - a_3 = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

As, difference of successive terms are equal,

So, 0, $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ … is an AP with common difference ¼.

Therefore, the next three term will be,

$\frac{3}{4} + \frac{1}{4}, \frac{3}{4} + 2(\frac{1}{4}), \frac{3}{4} + 3(\frac{1}{4})$

$1, \frac{5}{4}, \frac{3}{2}$

(ii) 5, $\frac{14}{3}, \frac{13}{3}$, 4…

$a_1 = 5$

$a_2 = \frac{14}{3}$

$a_3 = \frac{13}{3}$

$a_4 = 4$

$a_2 - a_1 = \frac{14}{3} - 5$

$= \frac{-1}{3}$

$a_3 - a_2 = \frac{13}{3} - \frac{14}{3}$

$= \frac{-1}{3}$

$a_4 - a_3 = 4 - \frac{13}{3}$

$= \frac{-1}{3}$

As, difference of successive terms are equal,

So, 5, $\frac{14}{3}$, $\frac{13}{3}$, 4… is an AP with common difference -1/3.

Hence, the next three term will be,

$4 + (\frac{-1}{3}), 4 + 2(\frac{-1}{3}), 4 + 3(\frac{-1}{3})$

$\frac{11}{3}, \frac{10}{3}, 3$

(iii)

$\sqrt{3}, 2\sqrt{3}, 3\sqrt{3},\ldots$

$a_1 = \sqrt{3}$

$a_2 = 2\sqrt{3}$

$a_3 = 3\sqrt{3}$

$a_4 = 4\sqrt{3}$

$a_2 - a_1 = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

$a_3 - a_2 = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$

$a_4 - a_3 = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$

As, difference of successive terms are equal,

So, $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3},\ldots$ is an AP with common difference $\sqrt{3}$.

Hence, the next three term will be,

$4\sqrt{3} + \sqrt{3}, 4\sqrt{3} + 2\sqrt{3}, 4\sqrt{3} + 3\sqrt{3}$

$5\sqrt{3}, 6\sqrt{3}, 7\sqrt{3}$

(iv)
$a + b, (a + 1) + b, (a + 1) + (b + 1), \dots$

$a_1 = a + b$

$a_2 = (a + 1) + b$

$a_3 = (a + 1) + (b + 1)$

$a_2 - a_1 = (a + 1) + b - (a + b) = 1$

$a_3 - a_2 = (a + 1) + (b + 1) - (a + 1) - b = 1$

As, difference of successive terms are equal,

So, $a + b, (a + 1) + b, (a + 1) + (b + 1), \dots$ is an AP with common difference 1.

Hence, the next three term will be,

$(a + 1) + (b + 1) + 1, (a + 1) + (b + 1) + 1(2), (a + 1) + (b + 1) + 1(3)$

$(a + 2) + (b + 1), (a + 2) + (b + 2), (a + 3) + (b + 2)$

(v) $a, 2a + 1, 3a + 2, 4a + 3,\dots$

$a_1 = a$

$a_2 = 2a + 1$

$a_3 = 3a + 2$

$a_4 = 4a + 3$

$a_2 - a_1 = (2a + 1) - (a)$
$= a + 1$

$a_3 - a_2 = (3a + 2) - (2a + 1)$
$= a + 1$

$a_4 - a_3 = (4a + 3) - (3a+2)$
$= a + 1$

As, difference of successive terms are equal, So, a, 2a + 1, 3a + 2, 4a + 3,… is an AP with common difference a+1.

Hence, the next three term will be,

4a + 3 +(a + 1), 4a + 3 + 2(a + 1), 4a + 3 + 3(a + 1)

5a + 4, 6a + 5, 7a + 6

3. Write the first three terms of the APs when *a* and *d* are as given below:

i. $a = \frac{1}{2}, d = -\frac{1}{6}$

ii. $a = -5, d = -3$

iii. $a = \sqrt{2}, d = \frac{1}{\sqrt{2}}$

Solution:

(i)

$a = \frac{1}{2}$, $d = -\frac{1}{6}$

First three terms of AP are :

a,

a + d,

a + 2d

$\frac{1}{2}, \frac{1}{2} + (-\frac{1}{6}), \frac{1}{2} + 2(-\frac{1}{6})$

$\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$

(ii)

a = –5, d = –3

First three terms of AP are:

a, a + d, a + 2d

-5, – 5 + 1 (- 3), – 5 + 2 (- 3)

– 5, – 8, – 11

(iii)

$a = \sqrt{2}$, $d = \frac{1}{\sqrt{2}}$

First three terms of AP are :

a, a + d, a + 2d

$\sqrt{2}, \sqrt{2}+\frac{1}{\sqrt{2}}, \sqrt{2}+\frac{2}{\sqrt{2}}$

$\sqrt{2}, \frac{3}{\sqrt{2}}, \frac{4}{\sqrt{2}}$

4. Find *a*, *b* and *c* such that the following numbers are in AP: *a*, 7, *b*, 23, *c*.

Solution:

To be a, 7, b, 23, c… in AP.

It should satisfy the condition,

$a_5 - a_4 = a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = d$ (as common difference is same)

$7 - a = b - 7 = 23 - b = c - 23$

So,

$b - 7 = 23 - b$

$2b = 30$

$b = 15$

Also,

$7 - a = b - 7$
$7 - a = 15 - 7$ (putting value of b)
$a = -1$

And,

$c - 23 = 23 - b$

$c - 23 = 23 - 15$

$c - 23 = 8$

$c = 31$

So,
$a = -1$

$b = 15$

c = 31

So, we can say that, the sequence – 1, 7, 15, 23, 31 is an AP

5. Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.

Solution:
As given in the question,
5th term,
$a_5 = 19$

Using the formula,
$a_n = a + (n - 1)d$
We have,
$a + 4d = 19$
$a = 19 - 4d$ …(1)

And,
20th term – 8th term = 20
$a + 19d - (a + 7d) = 20$
$12d = 20$
$d = \frac{4}{3}$

Putting $d = \frac{4}{3}$ in equation 1,

We get,

$a = 19 - 4(\frac{4}{3})$

$a = \frac{41}{3}$

The required AP is,
$\frac{41}{3}, \frac{41}{3} + \frac{4}{3}, \frac{41}{3} + 2(\frac{4}{3})$

$\frac{41}{3}, 15, \frac{49}{3}$

6. The 26th, 11th and the last term of an AP are 0, 3 and $-\frac{1}{5}$, respectively. Find the common difference and the number of terms.

Solution:

Given:

$a_{26} = 0$,

$a_{11} = 3$ and

$a_n = -\frac{1}{5}$

$a_{26} = 0$ [Given]

$a + (26 – 1)d = 0$

$a + 25d = 0$ ...(i)

$a_{11} = 3$ [Given]

$a + (11 – 1)d = 3$

$a + 10d = 3$ ...(ii)

$a_n = a + (n – 1)d = -\frac{1}{5}$ ...(iii)

On subtracting eqn. (ii) from eqn. (i), we get

$15d = –3$

$d = -\frac{1}{5}$

From (ii),

$a + 10d = 3$

$a – 2 = 3$

$a = 3 + 2$

$a = 5$

From (iii),

$a + (n – 1)d = -\frac{1}{5}$

$5 + (n – 1) \times -\frac{1}{5} = -\frac{1}{5}$

Multiplying both sides by 5, we get

$25 – (n – 1) = –1$

$25 + 1 = (n – 1)$

$n – 1 = 26$

$n = 27$

So, the common difference and number of terms in the A.P. are $-\frac{1}{5}$ and 27 respectively.

7. The sum of the 5^{th} and the 7^{th} terms of an AP is 52 and the 10^{th} term is 46. Find the AP.

Solution:
Let 1st term and common difference of an A.P be a and d
As given in the question,

$a_5 + a_7 = 52$
$a + (5 – 1)d + a + (7 – 1)d = 52$ (an = a + (n – 1)d)
$2a + 4d + 6d = 52$
$2a + 10d = 52$
$a + 5d = 26$ ...(i)

Also,
$a_{10} = 46$ (Given)
$a + (10 – 1)d = 46$
$a + 9d = 46$ ...(ii)

Subtracting (i) from (ii), we get,
$d = 5$

Now,
$a + 5d = 26$ (From (i))
$a + 5 \times 5 = 26$
$a = 26 – 25$
$a = 1$

A.P. is given by a, a + d, a + 2d, ...
So, the required A.P. is given by 1, 6, 11, 16, ...

8. Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first termbeing 12.

Solution:
Let us consider an A.P. with first term and common difference are ‘a’ and ‘d’.
We have,
$a_7 = a_{11} – 24$
$a + (7 – 1)d + 24 = a + (11 – 1)d$ [$a_n = a + (n – 1)d$]
$a + 6d + 24 – a = 10d$
$6d – 10d = –24$
$–4d = –24$
$d=6$

Now,
$a_n = a + (n – 1)d$
$a_{20} = 12 + (20 – 1)6$ [As ,n = 20, a = 12, d = 6]
$= 12 + 19 \times 6$
$= 12 + 114$
$a_{20} = 126$

So, the 20th term of the A.P. is 126.

9. If the 9^{th}term of an AP is zero, prove that its 29^{th}term is twice its 19^{th}term.

Solution:

Consider an A.P. whose first term and common difference are ‘a’ and ‘d’ respectively.
$a_9 = 0$ [Given]
$a + (9 – 1)d = 0$ [$a_n = a + (n – 1)d$]
$a + 8d = 0$
$a = –8d$ ...(i)

We have to prove that $a_{29} = 2a_{19}$
So, $a_{29} = a + (29 – 1)d$
$= –8d + 28d$ [Using equation (i)]
$a_{29} = 20d$ ...(ii)
Now,
$a_{19} = a + (19 – 1)d$
$a_{19} = –8d + 18d$ [Using (i)]
$a_{19} = 10d$

But, $a_{29} = 20d$ [From (ii)]
$= 2 \times 10d$
$= 2 \times a_{19}$ [$a_{19} = 10d$]
$= 2a_{19}$
$a_{29} = 2a_{19}$

Hence, the 29th term is twice the 19th term in the given A.P.

10. Find whether 55 is a term of the AP: 7, 10, 13,--- or not. If yes, find which term it is.

Solution:

55 will be nth term of the given A.P. if value of n is a natural number.
$a = 7$,
$d = 10 – 7$
$= 3$
Let 55 be the nth term of the given A.P.
$a_n = 55$ [assumed]
$7 + (n – 1)3 = 55$ [₹ $a_n = a +(n – 1)d$]
$(n – 1)3 = 55 – 7$
$n – 1 = 16$
$n = 17$, which is a natural number

So, 55 is the 17th term of the given A.P.

11. Determine *k* so that $k^2+4k+8, 2k^2+3k+6, 3k^2+4k+4$ are three consecutive terms of an AP.

Solution:

Since, $k^2+4k+8, 2k^2+3k+6, 3k^2+4k+4$ and $3k^2 + 4k + 4$ are consecutive terms of an AP.
$2k^2 + 3k + 6 - (k^2 + 4k + 8) = d$

$3k^2 + 4k + 4 - (2k^2 + 3k + 6) = d$

$2k^2 + 3k + 6 - k^2 - 4k - 8 = 3k^2 + 4k + 4 - 2k^2 - 3k - 6$

$k^2 - k - 2 = k^2 + k - 2$
$-k = k$
$2k = 0$
$k = 0$

12. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.

Solution:

We know that,
If the sum of three consecutive terms of an AP is given so terms can be considered as (a – d), a, (a + d).
Considering an A.P. whose three consecutive terms are (a – d), a, (a + d).
So,
$(a - d) + a + (a + d) = 207$
$3a = 207$
$a = 69$

Also, $(a - d)(a) = 4623$
$(69 - d)69 = 4623$ (a = 69)

$69 - d = 67$
$d = 69 - 67$
$d = 2$

So,
A.P. = (a – d), a, (a + d)
= (69 – 2), 69, (69 + 2)
= 67, 69, 71
Therefore, 207 can be divided into 67, 69, 71 which form three terms of an A.P.

13. The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.

Solution:
We know that,
Sum of interior angles of a triangle is 180°.
So, 180° is divided into three parts which are in A.P.
So, the terms of A.P. are (a – d), a, (a + d).

a – d + a + a + d = 180° [Angle sum property of a triangle]
3a = 180°
a = 60°

Also, the greatest angle is twice of the smallest. [Given]

a + d = 2(a – d)
a + d = 2a – 2d
a + d – 2a + 2d = 0
–a + 3d = 0
3d = a

Also, a = 60°
d = 20°

Required parts are a – d, a, a + d
= 60° – 20°, 60°, 60° + 20°
= 40°, 60°, 80°
Hence, the angles of the triangle are 40°, 60° and 80°.

14. If the *n*th terms of the two APs: $9, 7, 5, \ldots$ and $24, 21, 18, \ldots$ are the same, find the value of *n*. Also find that term.

Solution:
First A.P. is 9, 7, 5, ...
$a_1 = 9$,
d = 7 – 9
= –2
Now,
$a_n = a + (n - 1)d$
$= 9 + (n - 1)(-2)$
$= 9 - 2(n - 1)$
$= 9 - 2n + 2$
$a_n = 11 - 2n$

Second A.P. is 24, 21, 18, ...

$a_n = 24 + (n - 1)(-3)$
$= 24 - 3n + 3$
$= 27 - 3n$

We have,
11 – 2n = 27 – 3n
3n – 2n = 27 – 11
n = 16

So, 16th term of 1st A.P
$a_{16} = a_1 + (n – 1)d$

$a_{16} = 9 + (16 – 1)(–2)$
$= 9 – 2 \times 15 = 9 – 30$
$a_{16} = –21$

16th term of 2nd A.P.,
= 24 + (16 – 1) (–3)
= 24 – 15 × 3
= 24 – 45
= –21
So, the 16th terms of both A.P.s are equal to – 21.

15. If sum of the 3rd and the 8th terms of an AP is 7 and the sum of the 7th and the 14th terms is –3, find the 10th term.

Solution:

Taking 1st term and common difference 0f an A.P a and d, respectively.
According to the question,
$a_3 + a_8 = 7$ [Given]
a + (3 – 1)d + a + (8 – 1)d = 7 [∵ an = a + (n – 1)d]
a + 2d + a + 7d = 7
2a + 9d = 7 ...(i)

Also, $a_7 + a_{14} = –3$ [Given]
a + (7 – 1)d + a + (14 – 1)d = –3
a + 6d + a + 13d = –3
2a + 19d = –3 ...(ii)

Now, subtracting (i) from (ii), we get
d = –1
Now, 2a + 9d = 7 [Using (i)]
2a + 9(–1) = 7
2a = 7 + 9
a = 8

$a_{10} = a + (10 – 1)d$
$= 8 + 9(–1)$
$a_{10} = –1$
So, the 10th term of A.P. is –1.

16. Find the 12th term from the end of the AP: $-2, -4, -6, \ldots, -100$.

Solution:

Considering the given A.P. in reverse order
and finding the term.
i.e.,
–100 ... –6, –4, –2.

Now,
$a = -100$
$d = a_n + 1 - a_n$
$= -4 - (-6)$
$= -4 + 6$
$= 2$
$n = 12$

$a_n = a + (n - 1)d$
$a_{12} = -100 + (12 - 1)(2)$
$= -100 + 11 \times 2 = -100 + 22$
$a_{12} = -78$
Therefore, the 12th term from the last of A.P. –2, –4, –6, ... –100 is –78.

17. Which term of the AP: $53, 48, 43, \ldots$ **is the first negative term?**

Solution:

We have A.P. is 53, 48, 43,...
$a = 53$,
$d = 48 - 53$
$= -5$
Taking the nth term of A.P. is the first negative term.

Then, $a_n < 0$
$a + (n - 1)d < 0$
$53 + (n - 1)(-5) < 0$
$-5(n - 1) < -53$
$5(n - 1) > 53$
$5n - 5 > 53$
$5n > 53 + 5$
$n > 11.6$
$n = 12$
So, the first negative term of A.P. is 12th term,

$a_{12} = a + (n - 1)d$
$= 53 + (12 - 1)(-5)$
$= 53 - 5 \times 11$
$= 53 - 55$

= –2

18. How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3?

Solution:

The least number between 10 and 300 which leaves remainder 3 after dividing by 4 is 11.
The largest number between 10 and 300 which leaves remainder 3 on dividing by 4 is
296 + 3 =299.

So, 1st term or number = 11,
2nd term or number = 15
3rd term or number = 19,
last term or number = 299

A.P. becomes 11, 15, 19, ..., 299
$a_n = 299$,
a = 11,
d = 15 – 11
= 4,
n = ?

Now, a + (n – 1)d = 299
11 + (n – 1)4 = 299
(n – 1)4 = 299 – 11

n – 1 =72
n = 72 + 1
n = 73
Hence, the required numbers between 10 and 300, which when divided by 4 leave a remainder 3 are 73.

19. Find the sum of the two middle most terms of the AP: $-\frac{4}{3}, -1, -\frac{2}{3}, \ldots, 4\frac{1}{3}$.

Solution:

$a = \frac{-4}{3}$

$d = -1 + \frac{4}{3}$

$d = \frac{1}{3}$

Also,

$l = \frac{13}{3}$

$a + (n-1)d = \frac{13}{3}$

So,

$n = 18$

Middle most terms are:

$\frac{n}{2}$ th and $(\frac{n}{2}+1)$th

Which are,

$\frac{18}{2}$ term and $(\frac{18}{2}+1)$ term

that are,

9 th and 10 th terms,

So,

$a_9 = \frac{4}{3}$

$a_{10} = \frac{5}{3}$

Sum $= a_9 + a_{10}$

Sum $= 3$

20. The first term of an AP is –5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.

Solution:

Let the first term, common difference and the number of terms of an AP be a, d and n respectively.

Given that,

a = -5

l = 45

Sum of the terms of the AP = 120

$S_n = 120$

We know that, if last term of an AP is known, then sum of n terms of an AP is,

$S_n = \frac{n}{2}(a + l)$

$120 = \frac{n}{2}(-5 + 45)$

$120 \times 2 = 40 \times n$

$n = 3 \times 2$

$n = 6$

Number of terms of an AP is known, then the nth term of an AP is,

$l = a + (n - 1)d$

$45 = -5 + (6 - 1)d$

$50 = 5d$

$d = 10$

Hence, the common difference is 10.

So, number of terms and the common difference of an AP are 6 and 10 respectively.

21. Find the sum:

i. $1+(-2)+(-5)+(-8)+...+(-236)$

ii. $4-\frac{1}{n}+4-\frac{2}{n}+4-\frac{3}{n}+...$ **upto *n* terms.**

iii. $\frac{a-b}{a+b}+\frac{3a-2b}{a+b}+\frac{5a-3b}{a+b}+...$ **to 11 terms.**

Solution:

(i)

a = 1 and

d = (-2) – 1

= -3

Sum of n terms of an AP,

$S_n = \frac{n}{2}(2a+(n-1)d)$

$S_n = \frac{n}{2}(2\times1+(n-1)\times-3)$

$S_n = \frac{n}{2}(5-3n)$

We know that, if the last term (l) of an AP is known, then

$l = a + (n - 1)d$

$-236 = 1 + (n - 1)(-3)$ [$\because l = -236$, given]

$-237 = -(n - 1) \times 3$

$n - 1 = 79$

$n = 80$

Now, put the value of n in we get ,

$S_n = 40[5 - 3 \times 80]$
$= 40[5- 240]$
$= 40 \times (-235)$
$= -9400$
The required sum is -9400.

(ii)

$$a = 4 - \frac{1}{n}$$

$$d = (4 - \frac{2}{n}) - (4 - \frac{1}{n})$$

$$d = -\frac{1}{n}$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{7n-1}{2}$$

(iii)

$$a = \frac{a-b}{a+b}$$

$$d = \frac{3a-2b}{a+b} - \frac{a-b}{a+b}$$

$$d = \frac{2a-b}{a+b}$$

Also,

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right]$$

$$S_n = \frac{n}{2}(\frac{2an-bn-b}{a+b})$$

So,

$$S_{11} = \frac{11(11a-6b)}{a+b}$$

22. Which term of the AP: –2, –7, –12,... will be –77? Find the sum of this AP up to the term –77.

Solution:

Given, AP : -2, -7, -12, ….
Taking the nth term of an AP is -77
a = -2 and
d = -7 – (-2)

= -7 + 2
= -5

nth term of an AP,
$T_n = a + (n - 1)d$
$-77 = -2 + (n - 1)(-5)$
$-75 = -(n - 1) \times 5$
$(n - 1) = 15$
$n = 16$
So, the 16th term of the given AP will be -77.

Now, the sum of n terms of an AP is
$S_n = \frac{n}{2} [2a + (n - 1)d]$
So, sum of 16 terms i.e., upto the term -77.

$S_{16} = \frac{n}{2} [2 \times (-2) + (n - 1)(-5)]$
= 8[-4 + (16 – 1)(-5)]
= 8(-4 – 75)
= 8 × (-79)
= -632
Therefore, the sum of this AP upto the term -77 is -632.

23. If $a_n = 3 - 4n$, show that $a_1, a_2, a_3, \ldots$ form an AP. Also find S_{20}.

Solution:
Given that, nth term of the series is
$a_n = 3 - 4n$ …(i)

Putting n = 1,
$a_1 = 3 - 4(1)$
= 3 – 4
= -1

Putting n = 2,
$a_2 = 3 - 4(2)$
= 3 – 8
= -5

Putting n = 3,
$a_3 = 3 - 4(3)$
= 3 – 12
= -9

Putting n = 4,
$a_4 = 3 - 4(4)$
= 3 – 16

= -13
So, the series becomes -1, -5, -9, -13,….

We see that,
$a_2 – a_1$ = -5 – (-1)
= -5 + 1
= -4,

$a_3 – a_2$ = -9- (-5)
= -9 + 5
= -4,

$a_4 – a_3$ = -13 – (-9)
= -13 + 9
= -4

i.e., $a_2 – a_1 = a_3 – a_2 = a_4 – a_3 = … = -4$
Since, the each successive term of the series has the same difference. So, it forms an AP. We know that, sum of n terms of an AP,

$S_n = \frac{n}{2}$ [2a + (n – 1)d]

Sum of 20 terms of the AP,
S_{20} = 10[2(-1) + (20 – 1)(-4)]
= 10 [-2 + (19)(-4)]
= 10(-2 – 76)
= 10 × (-78) = -780
So, the required sum of 20 terms i.e., S_{20} is -780

24. In an AP, if $S_n = n(4n+1)$, find the AP.

Solution:

The nth term of an AP is
$a_n = S_n – S_n – 1$
a_n = n(4n + 1) – (n – 1)[4(n -1) + 1]

[as, S_n = n (4n + 1)]
an = $4n^2$ + n- (n – 1)(4 n – 3)
= $4n^2$ + n – 4n2 + 3n + 4n – 3
= 8n – 3
Put n = 1,
a_1 = 8(1) -3
= 5
Put n = 2,
a_2 = 8(2) – 3
= 16 – 3
= 13

Put n = 3,
$a_3 = 8(3) – 3$
$= 24 – 3$
$= 21$
So, the required AP is 5, 13, 21,….

25. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of *k*.

Solution:

We have, nth term of an AP,
$a_n = S_n – S_n – 1$
$= 3n^2 + 5n – 3(n – 1)^2 – 5(n – 1)$ [As $Sn = 3n^2 + 5n$ (given)]
$= 3n^2 + 5n – 3n^2 – 3 + 6n – 5n + 5$
$a_n = 6n + 2$ …………… (i)
$a_k = 6k + 2$
$= 164$ ($a_k = 164$ (given))
$6k = 164 – 2$
$= 162$

So,
k = 27

26. If S_n denotes the sum of first *n* terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$

Solution:

Sum of n terms of an AP $= \frac{n}{2}(2a + (n-1)d)$

Now,
$S_4 = 4a+6d$
$S_8=8a+28d$

So,
$S_8-S_4=4a+22d$
Now,
$S_{12}=\frac{12}{2}(2a+(n-1)d)$
$S_{12}=3(4a+22d)$
$S_{12} = 3(S_8 - S_4)$
Proved!!!

27. Find the sum of first 17 terms of an AP whose 4th and 9th terms are –15 and –30 respectively.

Solution:

Let us take the first term, common difference and the number of terms in an AP be a, d and n, respectively.
We know that, the nth term of an AP,

$T_n = a + (n – 1)d$ ….. (i)

4th term of an AP,

$T_4 = a + (4 – 1)d$
$= -15$ [given]
$a + 3d = -1$ ….(ii)

and 9th term of an AP,

$T_9 = a + (9 – 1)d = -30$ [given]
$a + 8d = -30$ …………. (iii)

Now, subtract Eq. (ii) from Eq. (iii), we get
$5d = -15$
$d = -3$
Put the value of d in Eq.(ii), we get
$a + 3(-3) = -15$
$a – 9 = -15$
$a = -15 + 9$
$= – 6$

Now putting values of a and d, we get,
$S_{17}= -510$

Hence, the required sum of first 17 terms of an AP is -510.

28. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

Solution:

Let a and d be the first term and common difference, of an AP.
Sum of n terms of an AP**,**

Now,
$S_6=36$
So,
$12=2a+5d$ ….1
Also,
$S_{16}=256$
So,
$32=2a+15d$ …..2

Subtracting eqn 1 and 2 we get,
d=2
a=1
Therefore putting value of and d in S_{10}, we get,
S_{10}=100

29. Find the sum of all the 11 terms of an AP whose middle most term is 30.

Solution:
As, the total number of terms (n) = 11 [odd]

Middle most term:

$$\frac{n+1}{2} term$$
$$= \frac{11+1}{2} term$$
$$= 6th\, term$$
$$Also,$$
$$a_6 = 30$$
$$a + 5d = 30$$
$$So,$$
$$S_{11} = \frac{n}{2}[2a + (11-1)d]$$
$$S_{11} = 11(a+5d)$$
$$S_{11} = 11 \times 30$$
$$S_{11} = 330$$

30. Find the sum of last ten terms of the AP: $8,10,12,...,126$**.**

Solution:

To find the sum of last ten terms, we write the given AP in reverse order.
i.e., 126, 124, 122,…., 12, 10, 8

a = 126,
d = 124 – 126
= – 2

$S_{10} = \frac{n}{2}$ [2a + (10 – 1)d]

As,

$S_n = \frac{n}{2}$ [2a + (n – 1)d]]

= 5{2(126) + 9(-2)}
= 5(252 – 18)
= 5 × 234
= 1170

31. Find the sum of first seven numbers which are multiples of 2 as well as of 9.
[Hint: Take the LCM of 2 and 9]

Solution:

To find the sum of first seven numbers which are multiples of 2 as well as of 9.
We take LCM of 2 and 9 which is 18.
Hence, the series becomes 18, 36, 54 ….

a = 18,
d = 36 – 18
= 18
Using the formula of S_n,

$$S_7 = \frac{7}{2}[2\times 18 + (7-1)18]$$

$$S_7 = \frac{7}{2}[36 + 6\times 18]$$

$$S_7 = 504$$

32. How many terms of the AP: $-15, -13, -11, \ldots$ are needed to make the sum –55? Explain the reason for double answer.

Solution:

Let we assume n number of terms are needed to make the sum -55
a= -15,
d = -13 + 15 = 2
Sum of n terms of an AP,

$$Sn = \frac{n}{2}[2a + (n-1)d]$$

$$-55 = \frac{n}{2}[2(-15) + (n-1)2]$$

Also,
Sn = -55 (given)

-55 = -15M + n(n – 1)
$n^2 - 16n + 55 = 0$
$n^2 - 11n - 5n + 55 = 0$
$n(n - 11) - 5(n - 11) = 0$
$(n - 11)(n - 5) = 0$

n = 5, 11

Either 5 or 11 terms are needed to make the sum -55 when n = 5,
AP will be -15, -13, -11, -9, -7,

So, resulting sum will be -55 because all terms are negative.
When n = 11,
AP will be -15, -13, -11, -9, -7, -5, -3, -1, 1, 3, 5

Hence, resulting sum will be -55 because the sum of terms 6th to 11th is zero.

33. The sum of the first *n* terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first 2*n* terms of another AP whose first term is – 30 and the common difference is 8. Find *n*.

Solution:
Given,
a = 8
d = 20

Let the number of terms in first AP be n.
Sum of first n terms of an AP,

$$S_n = \frac{n}{2}[2\times 8+(n-1)20]$$

$$S_{31} = \frac{n}{2}(20n-4)$$

$$S_{31} = n(10n-2)$$

Now,
first term of the second AP(a’) = – 30
Common difference of the second AP(d’) = 8

Sum of first 2n terms of second AP,

$$S_{2n} = \frac{2n}{2}[2a' + (2n-1)d']$$

S_{2n} = n[2(-30)+(2n – 1)(8)]
S_{2n} = n[-60 + 16n – 8)]
S_{2n} = n[16n – 68]

Now, by given condition,
Sum of first n terms of the first AP = Sum of first 2n terms of the second AP

S_n — S_{2n}

n(10n – 2) = n(16n – 68)
n[(16n – 68) – (10n – 2)] = 0

n(16n – 68 – 10n + 2) = 0
n(6n – 66) = 0
n = 11

So, the required value of n is 11.

34. Kanika was given her pocket money on Jan 1st, 2008. She puts Re 1 on Day 1,Rs 2 on Day 2, Rs 3 on Day 3, and continued doing so till the end of the month, from this money into her piggy bank. She also spent Rs 204 of her pocket money, and found that at the end of the month she still had Rs 100 with her. How much was her pocket money for the month?

Solution:

Let her pocket money be ₹ x
If, she puts 11 on day 1, ₹ 2 on day 2, ₹ 3 on day 3 and so on till the end of the month, from this money into her piggy bank.
So,
1 + 2 + 3 + 4 + … + 31
which form an AP in which terms are 31
a = 1,
d = 2 – 1
= 1
Sum of first 31 terms is S_{31}

$$S_{31} = \frac{31}{2}[2\times1+(31-1)1]$$
$$S_{31} = \frac{31\times32}{2}$$
$$S_{31} = 496$$

Hence, Kanika takes ₹ 496 till the end of the month from this money.

Also, she spent ₹ 204 of her pocket money and found that at the end of the month she still has ₹ 100 with her.
So,
(x – 496) – 204 = 100
x – 700 = 100
x = ₹ 800
Therefore, ₹ 800 was her pocket money for the month.

35. Yasmeen saves Rs 32 during the first month, Rs 36 in the second month and Rs 40 in the third month. If she continues to save in this manner, in how many months will she save Rs 2000?

Solution:

Yasmeen, during the first month, saves = ₹ 32

During the second month, saves = ₹ 36

During the third month, saves = ₹ 40

Let we take Yasmeen saves Rs 2000 during the n months.

So, we have arithmetic progression 32, 36, 40…

$a = 32$,

$d = 36 – 32$

$= 4$

and she saves total money,

S_n = ₹ 2000

We know that, sum of first n terms of an AP is,

$S_n = \frac{n}{2}][2a + (n – 1)d]$

$2000 = \frac{n}{2}[2 \times 32 + (n -1) \times 4]$

$2000 = n(32 + 2n – 2)$

$2000 = n(30 + 2n)$

$1000 = n (15 + n)$

$1000 = 15n + n^2$

$n^2 + 15n – 1000 = 0$

$n^2 + 40n – 25n – 1000 = 0$

$n(n + 40) – 25(n + 40) = 0$

$(n + 40)(n – 25)= 0$

$n = 25$

$n \neq – 40$

[As, months cannot be negative]

So, in 25 months she will save ₹ 2000.