Exercise No. 5.2

Short Answer Questions with Reasoning:

Question:

1. Which of the following form an AP? Justify your answer.

i. $-1, -1, -1, -1, \ldots$

ii. $0, 2, 0, 2, \ldots$

iii. $1, 1, 2, 2, 3, 3, \ldots$

iv. $11, 22, 33, \ldots$

v. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

vi. $2, 2^2, 2^3, 2^4, \ldots$

vii. $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

Solution:

(i) –1, –1, –1, –1,…

We have

$a_1 = -1$,

$a_2 = -1$,

$a_3 = -1$ and $a_4 = -1$

$a_2 - a_1 = 0$

$a_3 - a_2 = 0$

$a_4 - a_3 = 0$

As the difference of successive terms is same, therefore given list of numbers from an AP.

(ii) 0, 2, 0, 2,…

We have

$a_1 = 0$,

$a_2 = 2$,

$a_3 = 0$ and $a_4 = 2$

$a_2 - a_1 = 2$

$a_3 - a_2 = -2$

$a_4 - a_3 = 2$

The difference of successive terms is not same, therefore given list of numbers does not form an AP.

(iii) 1, 1, 2, 2, 3, 3…

We have

$a_1 = 1$,

$a_2 = 1$,

$a_3 = 2$ and $a_4 = 2$

$a_2 - a_1 = 0$

$a_3 - a_2 = 1$

The difference of successive terms is not same, therefore given list of numbers does not form an AP.

(iv) 11, 22, 33…

We have

$a_1 = 11$,

$a_2 = 22$ and $a_3 = 33$

$a_2 - a_1 = 11$

$a_3 - a_2 = 11$

The difference of successive terms is same, therefore given list of numbers form an AP.

(v) 1/2,1/3,1/4, …

We have

$a_1 = ½$,

$a_2 = 1/3$ and $a_3 = 1/4$

$a_2 - a_1 = -1/6$

$a_3 - a_2 = -1/12$

The difference of successive terms is not same, therefore given list of numbers does not form an AP.

(vi) $2, 2^2, 2^3, 2^4, \ldots$

We have

$a_1 = 2$,

$a_2 = 2^2$,

$a_3 = 2^3$ and $a_4 = 2^4$

$a_2 - a_1 = 2^2 - 2$

$= 4 - 2$

$= 2$

$a_3 - a_2 = 2^3 - 2^2$

$= 8 - 4$

$= 4$

The difference of successive terms is not same, therefore given list of numbers does not form an AP.

(vii) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

We have,

$a_1 = \sqrt{3}$,

$a_2 = \sqrt{12}$,

$a_3 = \sqrt{27}$ and $a_4 = \sqrt{48}$

$a_2 - a_1 = \sqrt{12} - \sqrt{3}$

$= 2\sqrt{3} - \sqrt{3}$

$= \sqrt{3}$

$a_3 - a_2 = \sqrt{27} - \sqrt{12}$

$= 3\sqrt{3} - 2\sqrt{3}$

$= \sqrt{3}$

$a_4 - a_3 = \sqrt{48} - \sqrt{27}$

$= 4\sqrt{3} - 3\sqrt{3}$

$= \sqrt{3}$

The difference of successive terms is same, therefore given list of numbers from an AP.

2. Justify whether it is true to say that $-1, -\frac{3}{2}, -2, \frac{5}{2}, \ldots$ forms an AP as $a_2 - a_1 = a_3 - a_2$.

Solution:

It is not true.

$a_1 = -1$,

$a_2 = \frac{-3}{2}$,

$a_3 = -2$

$a_4 = \frac{5}{2}$

$a_2 - a_1 = \frac{-3}{2} - (-1)$

$= \frac{-1}{2}$

$a_3 - a_2 = -2 - (\frac{-3}{2})$

$= \frac{-1}{2}$

$a_4 - a_3 = \frac{5}{2} - (-2)$

$= \frac{9}{2}$

As, the difference of successive terms in not same, all though, $a_2 - a_1 = a_3 - a_2$

but $a_4 - a_3 \neq a_3 - a_2$ so, it does not form an AP.

3. For the AP: $-3, -7, -11, \ldots,$ can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20}? Give reasons for your answer.

Solution:

It is true.

We have, a = – 3

$d = a_2 - a_1$

$= -7 - (-3)$

$= -4$

$a_{30} - a_{20} = a + 29d - (a + 19d)$

$= 10d$

$= -40$

Therefore, difference between any two terms of an AP is proportional to common difference of that AP.

4. Two APs have the same common difference. The first term of one AP is 2 and that of the other is 7. The difference between their 10th terms is the same as the difference between their 21st terms, which is the same as the difference between any two corresponding terms. Why?

Solution:

Let us assume, there are two AP's with first terms a and their common differences are d and D respectively.

Taking n be any term,

$a_n = a + (n – 1)d$

$A_n = A + (n – 1)D$

As common difference is equal for both AP's

We have D = d

So, we have

$A_n – a_n = a + (n – – 1)d – [A + (n – 1)D]$

$= a + (n – 1)d – A – (n – 1)d$

$= a – A$

Since, a – A is a constant value.

So, difference between any corresponding terms will be equal to a – A.

5. Is 0 a term of the AP: $31, 28, 25, \ldots$ **? Justify your answer.**

Solution:

We know, an = a + (n – 1)d

If we put the values of a_n, a, and d in the above equation and if n comes out to be a natural number then, the given a_n will be the term of the given series.

$a_n = 0$, a = 31

$d_1 = 28 – 31$

$= –3$,

$d_2 = 25 – 28$

$= –3$

Hence,

$d_1 = d_2 = –3$

$a_n = a + (n – 1)d$

$0 = 31 + (n – 1) (–3)$

$-31 = -(n-1) \times 3$

$(n-1) = \frac{31}{3}$

As n is in fraction and is not a natural number so 0 (an) is not any term of the given A.P.

6. The taxi fare after each km, when the fare is Rs 15 for the first km and Rs 8 for each additional km, does not form an AP as the total fare (in Rs) after each km is $15, 8, 8, 8, \ldots$.
Is the statement true? Give reasons.

Solution:

No, the given statement is false.

15, 8, 8, 8 ... are not the total fare for 1, 2, 3, 4, km respectively.

Total fare for Ist km = Rs 15.

Total fare for 2 km = Rs 15 + Rs 8

= Rs 23

Total fare for 3 km = Rs 23 + Rs 8

= Rs 31

Total fare for 4 km = Rs 31 + Rs 8

= Rs 39

Total fare for 1 km, 2 km, 3km, 4km, ... are Rs15, Rs 23, Rs 31, Rs 39, ... respectively.

Now,

$d_1 = 23 - 15$

$= 8$

$d_2 = 31 - 23$

$= 8$

$d_3 = 39 – 31$

$= 8$

Therefore, the total fare for 1 km, 2 km, 3km, 4km, ...from an A.P. as 15, 23, 31, 39, ...

And, fare for each km does not form A.P. as 15, 8, 8, 8,...

7. In which of the following situations, do the lists of numbers involved form an AP?Give reasons for your answers.

i. The fee charged from a student every month by a school for the whole session,when the monthly fee is Rs 400.

ii. The fee charged every month by a school from Classes I to XII, when the monthly fee for Class I is Rs 250, and it increases by Rs 50 for the next higher class.

iii. The amount of money in the account of Varun at the end of every year when Rs 1000 is deposited at simple interest of 10% per annum.

iv. The number of bacteria in a certain food item after each second, when they double in every second.

Solution:

(i)

The school charges from a student every month fees =₹400.

So, the fee charged from a student in the whole session is 400, 400, 400, 400, ...

As

$d_1 = d_2 = d_3 = d_{12} = 0$ so, the series of numbers is an A.P.

(ii)

Fee for 1st class = ₹250

Fee for 2nd class = ₹ (250 + 50)

= ₹ 300

Fee for 3rd class = ₹ (300 + 50)

= ₹ 350

Fee for 4rth class = ₹ (350 + 50)

= ₹ 400

So, 250, 300, 350, 400,... is a series consisting of 12 terms.

$d_1 = 300 - 250 =$ ₹ 50,

$d_2 = 350 - 300 =$ ₹ 50,

$d_3 = 400 - 350 =$ ₹ 50

$d_1 = d_2 = d_3 =$ ₹ 50

So, the list of numbers 250, 300, 350, 400 ... is in A.P.

(iii)

₹100

So, ₹100 is credited at the end of each year in the account of Varun.

Money in the beginning of 1st year (deposited) = ₹ 1000

Money at the end of 1st year when interest credited = 1000 + 100

= ₹ 1100

Money at the end of 2nd year = 1100 + 100

= ₹ 1200

Money at the end of 3rd year = 1200 + 100

= ₹1300

Money at the end 4th year = 1300 + 100

= ₹1400

So, Amount of money at the end of each year starting initially from Ist year is given by

1000, 1100, 1200, 1300, 1400...

Also,

$d_1 = d_2 = d_3 = d_4 = 100$

So, the list of numbers is an A.P.

(iv)

Taking the number of bacteria present initially = x

So, the number of bacteria preset after 1 sec = 2(x) = 2x

Number of bacteria present after 2 seconds = 2(2x) = 4x

Number of bacteria present after 3 seconds = 2(4x) = 8x

Number of bacteria present after 4 seconds = 2(8x) = 16x

Hence, the number of bacteria from starting to end of each second are given by x, 2x, 4x, 8x, 16x,….

Now,

d_1= 2x – x

= x,

d^2= 4x – 2x

= 2x

Also, $d_1 \neq d_2$

Hence, the list of numbers does not from an A.P.

8. Justify whether it is true to say that the following are the *n*th terms of an AP.

i. $2n-3$

ii. $3n^2+5$

iii. $1+n+n^2$

Solution:

(i) $a_n = 2n - 3$
a_1= 2(1) – 3
= 2 – 3
= –1,

a_2= 2(2) – 3
= 4 – 3
= 1

a_3= 2(3) – 3
= 6 – 3
= 3,

a_4= 2(4) – 3

= 8 – 3
= 5

Also,
d_1= 1 – (–1)
= 1 + 1
= 2,

d_2= 3 – 1
= 2,

d_3= 5 – 3
= 2

d_1= d_2= d_3= 2,
Hence, an = 2n – 3 is the nth term of an A.P.

(ii)
$a_n = 3n^2 + 5$

$a_1 = 3(1)^2 + 5$
= 3 × 1 + 5
= 3 + 5 = 8

$a_2 = 3(2)^2 + 5$
= 3 × 4 + 5
= 12 + 5 = 17

$a_3 = 3(3)^2 + 5$
= 3 × 9 + 5
= 27 + 5 = 32

$a_4 = 3(4)^2 + 5$
= 3 × 16 + 5
= 48 + 5 = 53

$a_5 = 3(5)^2 + 5$
= 3 × 25 + 5
= 75 + 5 = 80

∴ $d_1 = a_2 – a_1$
= 17 – 8 = 9,

$d_2 = a_3 – a_2$
= 32 – 17 = 15

$d_3 = a_4 – a_3$
= 53 – 32 = 21,

$d_4 = a_5 - a_4$
$= 80 – 53 = 27$

As, $d_1 \neq d_2$
Hence, an = $3n^2 + 5$ is not the nth term of an A.P.

(iii) $a_n = 1 + n + n^2$

$a_1 = 1 + (1) + (1)^2$

$= 1 + 1 + 1 = 3$

$a_2 = 1 + (2) + (2)^2$

$= 1 + 2 + 4 = 7$

$a_3 = 1 + (3) + (3)^2$

$= 1 + 3 + 9 = 13$

$a_4 = 1 + (4) + (4)^2$

$= 1 + 4 + 16 = 21$

$a_5 = 1 + (5) + (5)^2$

$= 1 + 5 +25 = 31$

So,
$d_1 = a_2 - a_1$
$= 7 – 3 = 4$
$d_2 = a_3 - a_2$
$= 13 – 7 = 6$
$d_3 = a_4 - a_3$
$= 21 – 13 = 8$

$d_4 = a_5$ -,a_4
$= 31 – 21 = 10$

As d1≠ d2
Hence, $a_n = 1 + n + n^2$ is not the nth term of an A.P.