Exercise 13.2

Question 1.
Using laws of exponents, simplify and write the answer in exponential form:
(i) $3^2 \times 3^4 \times 3^8$

Solution:-
By Using the rule of 'Multiplying the powers with same base:
$a^m \times a^n = a^{m+n}$
Then,
$= (3)^{2+4+8}$
$= 3^{14}$

(ii) $6^{15} \div 6^{10}$

Solution:-
By Using the rule of 'Dividing the powers with same base:
$a^m \div a^n = a^{m-n}$
Then,
$= (6)^{15-10}$
$= 6^5$

(iii) $a^3 \times a^2$

Solution:-
By Using the rule of 'Multiplying the powers with same base:
$a^m \times a^n = a^{m+n}$
Then,
$= (a)^{3+2}$
$= a^5$

(iv) $7^x \times 7^2$

Solution:-
By Using the rule of 'Multiplying the powers with same base:
$a^m \times a^n = a^{m+n}$
Then,
$= (7)^{x+2}$

(v) $(5^2)^3 \div 5^3$

Solution:-

By Using the rule of taking power of as power $(a^m)^n = a^{mn}$, the given expression $(5^2)^3$ can be written as,

$= (5)^{2 \times 3}$

$= 5^6$

Now,

$5^6 \div 5^3$

By Using the rule of 'Dividing the powers with same base: $a^m \div a^n = a^{m-n}$. Then,

$= (5)^{6-3}$

$= 5^3$

(vi) $2^5 \times 5^5$

Solution:-

By Using the rule of 'Multiplying the powers with same exponents: $a^m \times b^m = ab^m$. Then,

$= (2 \times 5)^5$

$= 10^5$

(vii) $a^4 \times b^4$

Solution:-

Using the rule of 'Multiplying the powers with same exponents: $a^m \times b^m = ab^m$. Then,

$= (a \times b)^4$

$= ab^4$

(viii) $(3^4)^3$

Solution:-

Using the rule of taking power of as power: $(a^m)^n = a^{mn}$.

$(3^4)^3$ can be written as,

$= (3)^{4 \times 3}$

$= 3^{12}$

(ix) $(2^{20} \div 2^{15}) \times 2^3$

Solution:-

Using the rule of 'Dividing the powers with same base: $a^m \div a^n = a^{m-n}$, the given expression $(2^{20} \div 2^{15})$ can be simplified as,

$= (2)^{20-15}$

$= 2^5$

Then,

By Using the rule of 'Multiplying the powers with same base: $a^m \times a^n = a^{m+n}$,

$2^5 \times 2^3$ can be simplified as,

$= (2)^{5+3}$

$= 2^8$

(x) $8^t \div 8^2$

Solution:-

By Using the rule of 'Dividing the powers with same base: $a^m \div a^n = a^{m-n}$.

Then,

$= (8)^{t-2}$

Question 2.

Simplify and express each of the following in exponential form:

(i) $\frac{2^3 \times 3^4 \times 4}{3 \times 32}$

Solution:-

Factors of 32 = $2 \times 2 \times 2 \times 2 \times 2$

$= 2^5$

Factors of 4

$= 2 \times 2$

$= 2^2$

Then,

$= \frac{2^3 \times 3^4 \times 2^2}{3 \times 2^5}$

$= \frac{2^{3+2} \times 3^4}{3 \times 2^5}$ …… $[\because a^m \times a^n = a^{m+n}]$

$= \frac{2^5 \times 3^4}{3 \times 2^5}$

$= 2^{5-5} \times 3^{4-1}$ …… $[\because a^m \div a^n = a^{m-n}]$

$= 2^0 \times 3^3$

$= 1 \times 3^3$

$= 3^3$

(ii) $((5^2)^3 \times 5^4) \div 5^7$

Solution:-

$(5^2)^3$ can be written as

$= (5)^{2 \times 3}$ …... $[\because (a^m)^n = a^{mn}]$

$= 5^6$

Then,

$= (5^6 \times 5^4) \div 5^7$

$= (5^{6+4}) \div 5^7$ ..…. $[\because a^m \times a^n = a^{m+n}]$

$= 5^{10} \div 5^7$

$= 5^{10-7}$ …... $[\because a^m \div a^n = a^{m-n}]$

$= 5^3$

(iii) $25^4 \div 5^3$

Solution:-

$(25)^4$ can be written as

$= (5 \times 5)^4$

$= (5^2)^4$

$(5^2)^4$ can be written as

$= (5)^{2 \times 4}$ …… $[\because (a^m)^n = a^{mn}]$

$= 5^8$

Then,

$= 5^8 \div 5^3$

$= 5^{8-3}$ …... $[\because a^m \div a^n = a^{m-n}]$

$= 5^5$

(iv) $\frac{3 \times 7^2 \times 11^8}{21 \times 11^3}$

Solution:-

Factors of 21

$= 7 \times 3$

Then,

$= \frac{3 \times 7^2 \times 11^8}{7 \times 3 \times 11^3}$

$= 3^{1-1} \times 7^{2-1} \times 11^{8-3}$

$= 3^0 \times 7 \times 11^5$

$= 1 \times 7 \times 11^5$

$= 7 \times 11^5$

(v) $\frac{3^7}{3^4 \times 3^3}$

Solution:-

$= \frac{3^7}{3^{4+3}}$ …… $[\because a^m \times a^n = a^{m+n}]$

$= \frac{3^7}{3^7}$

$= 3^{7-7}$ …… $[\because a^m \div a^n = a^{m-n}]$

$= 3^0$

$= 1$

(vi) $2^0 + 3^0 + 4^0$

Solution:-

$= 1 + 1 + 1$

$= 3$

(vii) $2^0 \times 3^0 \times 4^0$

Solution:-

$= 1 \times 1 \times 1$

$= 1$

(viii) $(3^0 + 2^0) \times 5^0$

Solution:-

$= (1 + 1) \times 1$

$= (2) \times 1$

$= 2$

(ix) $\frac{2^8 \times a^5}{4^3 \times a^3}$

Solution:-

$(4)^3$ can be written as

$= (2 \times 2)^3$

$= (2^2)^3$

$(5^2)^4$ can be written as

$= (2)^{2 \times 3}$ …… $[\because (a^m)^n = a^{mn}]$

$= 2^6$

Then,

$= \frac{2^8 \times a^5}{2^6 \times a^3}$

$= 2^{8-6} \times a^{5-3}$ …… $[\because a^m \div a^n = a^{m-n}]$

$= 2^2 \times a^2$

$= 2a^2$ …… $[\because (a^m)^n = a^{mn}]$

(x) $\frac{a^5}{a^3} \times a^8$

Solution:-

$= (a^{5-3}) \times a^8$ …... $[\because a^m \div a^n = a^{m-n}]$

$= a^2 \times a^8$

$= a^{2+8}$ …… $[\because a^m \times a^n = a^{m+n}]$

$= a^{10}$

(xi) $(4^5 \times a^8 b^3)/(4^5 \times a^5 b^2)$

Solution:-

$= 4^{5-5} \times (a^{8-5} \times b^{3-2})$ …… $[\because a^m \div a^n = a^{m-n}]$

$= 4^0 \times (a^3 b)$

$= 1 \times a^3b$
$= a^3b$

(xii) $(2^3 \times 2)^2$

Solution:-
$= (2^{3+1})^2$ …… $[\because a^m \times a^n = a^{m+n}]$
$= (2^4)^2$
$(2^4)^2$ can be written as
$= (2)^{4 \times 2}$ …… $[\because (a^m)^n = a^{mn}]$
$= 2^8$

Question 3.
Say true or false and justify your answer:
(i) $10 \times 10^{11} = 100^{11}$

Solution:-
Considering:
Left Hand Side (LHS) $= 10 \times 10^{11}$
$= 10^{1+11}$ … $[\because a^m \times a^n = a^{m+n}]$
$= 10^{12}$
And, Considering
Right Hand Side (RHS) $= 100^{11}$
$= (10 \times 10)^{11}$
$= (10^{1+1})^{11}$
$= (10^2)^{11}$
$= (10)^{2 \times 11}$ …… $[\because (a^m)^n = a^{mn}]$
$= 10^{22}$
Comparing both LHS and RHS,
LHS ≠ RHS
Therefore, the given statement is false.

(ii) $2^3 > 5^2$

Solution:-
Considering
LHS $= 2^3$
Expansion of
$2^3 = 2 \times 2 \times 2$
$= 8$
And, Considering
RHS $= 5^2$
Expansion of
$5^2 = 5 \times 5$

= 25
Comparing both LHS and RHS,
LHS < RHS
$2^3 < 5^2$
Therefore, the given statement is false.

(iii) $2^3 \times 3^2 = 6^5$

Solution:-
Considering
LHS = $2^3 \times 3^2$
Expansion of
$2^3 \times 3^2 = 2 \times 2 \times 2 \times 3 \times 3$
= 72
And, Considering
RHS = 6^5
Expansion of
$6^5 = 6 \times 6 \times 6 \times 6 \times 6$
= 7776
Comparing both LHS and RHS,
LHS < RHS
$2^3 < 5^2$
Therefore, the given statement is false.

(iv) $3^0 = (1000)^0$

Solution:-
Considering
LHS = 3^0
= 1
And, Considering
RHS = 1000^0
= 1
Comparing both LHS and RHS,
LHS = RHS
$3^0 = 1000^0$
Therefore, the given statement is true.

Question 4.
Express each of the following as a product of prime factors only in exponential form:
(i) 108 × 192

Solution:-

Simplifying the value to find factors of 108 = 2 × 2 × 3 × 3 × 3
= $2^2 \times 3^3$
Simplifying the value to find factors of 192 = 2 × 2 × 2 × 2 × 2 × 2 × 3
= $2^6 \times 3$
Then,
= $(2^2 \times 3^3) \times (2^6 \times 3)$
= $2^{2+6} \times 3^{3+1}$ … [$\because a^m \times a^n = a^{m+n}$]
= $2^8 \times 3^4$

(ii) 270

Solution:-
Simplifying the value to find factors of 270 = 2 × 3 × 3 × 3 × 5
= $2 \times 3^3 \times 5$

(iii) 729 × 64

Solution:-
Simplifying the value to find factors of 729 = 3 × 3 × 3 × 3 × 3 × 3
= 3^6
Simplifying the value to find factors of 64 = 2 × 2 × 2 × 2 × 2 × 2
= 2^6
Then,
= $(3^6 \times 2^6)$
= $3^6 \times 2^6$

(iv) 768

Solution:-
Simplifying the value to find factors of 768 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3
= $2^8 \times 3$

Question 5.
Simplify:
(i) $\frac{(2^5)^2 \times 7^3}{8^3 \times 7}$

Solution:-
8^3 can be written as
= $(2 \times 2 \times 2)^3$
= $(2^3)^3$
We have,
= $\frac{(2^5)^2 \times 7^3}{(2^3)^3 \times 7}$

$= \frac{2^{5\times2}\times7^3}{2^{3\times3}\times7}$ …… [$\because (a^m)^n = a^{mn}$]

$= \frac{\mathbf{2^{10}\times7^3}}{\mathbf{2^9\times7}}$

$= (2^{10-9} \times 7^{3-1})$ ……[$\because a^m \div a^n = a^{m-n}$]

$= 2 \times 7^2$

$= 2 \times 7 \times 7$

$= 98$

(ii) $\frac{\mathbf{25\times5^2\times t^8}}{\mathbf{10^3\times t^4}}$

Solution:-

25 can be written as

$= 5 \times 5$

$= 5^2$

10^3 can be written as

$= 10^3$

$= (5 \times 2)^3$

$= 5^3 \times 2^3$

We have,

$= \frac{5^2\times5^2\times t^8}{5^3\times2^3\times t^4}$

$= \frac{5^{2+2}\times t^8}{5^3\times2^3\times t^4}$ …… [$\because a^m \times a^n = a^{m+n}$]

$= \frac{5^4\times t^8}{5^3\times2^3\times t^4}$

$= \frac{5^{4-3}\times t^{8-4}}{2^3}$ …… [$\because a^m \div a^n = a^{m-n}$]

$= \frac{5\times t^4}{2\times2\times2}$

$= \frac{5t^4}{8}$

(iii) $\frac{\mathbf{3^5\times10^5\times25}}{\mathbf{5^7\times6^5}}$

Solution:-

10^5 can be written as

$= (5 \times 2)^5$

$= 5^5 \times 2^5$

25 can be written as

$= 5 \times 5$

$= 5^2$

6^5 can be written as

$= (2 \times 3)^5$

$= 2^5 \times 3^5$

Then we have,

$= \frac{3^5 \times 5^5 \times 2^5 \times 5^2}{5^7 \times 2^5 \times 3^5}$

$= \frac{3^5 \times 5^{5+2} \times 2^5}{5^7 \times 2^5 \times 3^5}$ … [$\because a^m \times a^n = a^{m+n}$]

$= \frac{3^5 \times 5^7 \times 2^5}{5^7 \times 2^5 \times 3^5}$

$= (3^{5-5} \times 5^{7-7} \times 2^{5-5})$

$= (3^0 \times 5^0 \times 2^0)$ … [$\because a^m \div a^n = a^{m-n}$]

$= 1 \times 1 \times 1$

$= 1$