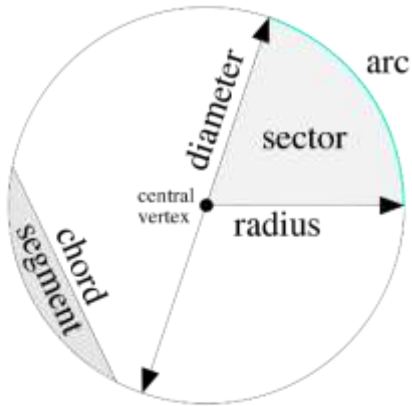
Circles



A chord of a circle is a straight-line segment whose endpoints both lie on the circle. A line that joins two points on the circumference of a circle is called a chord.

A chord that passes through a circle's centre point is the circle's diameter.

Every diameter is a chord, but not every chord is a diameter. We shall now see few theorems related to the chords.

Chord Properties of Circles

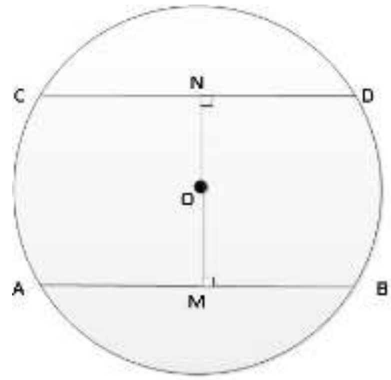
1. A circle is a locus of a point which moves in a plane in such a way that its distance from a fixed point in the same plane always remains a constant.
    1. The fixed point is called the centre of the circle.
    2. The constant distance is called the radius of the circle.
    3. The perimeter of the circle is called its circumference.

2. The line segment joining any two points on a circle is called a chord of the circle.
A chord of a circle passing through its centre is called a diameter of the circle. Thus, length of diameter = 2 x Radius.

Theorem related to equal chords

1. If O is the centre of the circle and chord AB = chord CD then OM = ON (Equal chord of a circle is equidistant from the centre).

2. If O is the centre of the circle and OM = ON then chord AB = chord CD (Chords which are equidistant from the centre are equal).



Theorem: A line drawn from the centre of a circle to bisect a chord is perpendicular to the chord.

If AB is a chord of a circle with centre O. AN = BN and ON is joined then ON is perpendicular to AB.

Theorem: The perpendicular from the centre to chord bisect the chord.

If AB is a chord of a circle with centre O. ON is perpendicular to AB.

Then AN = BN.

Theorem: There is one and only one circle passing through three given non-collinear points.

A, B and C are three non-collinear points.

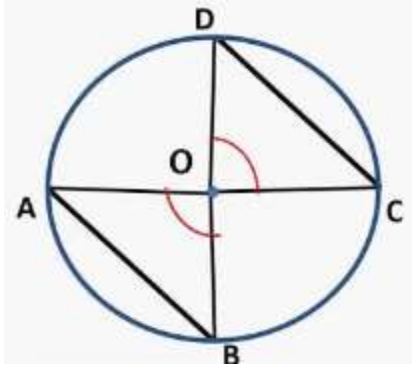
One and only one circle will pass through A, B and C.

Arc and its types

- An arc is a part of the circumference of a circle
    - The smaller arc is called the minor arc
    - The larger arc is called the major arc
- A segment is the part of the circle bounded by an arc and a chord.
    - Just like arcs, the segments are also of two types, major and minor segment

Theorem related to congruent arcs

- If chord AB = chord CD, then arc AB = arc CD.
- If arc AB = arc CD, then chord AB = chord CD.



Angle subtended by arcs at the centre and at the circumference.

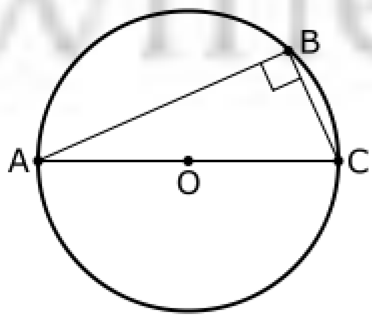
If O is the centre of the circle and A is any point on its circumference then ∠AOB = 2∠ACB.

Angles subtended by equal segments

1. Angles in the same segment of a circle are equal
2. ∠PAQ = ∠PCQ.
3. ∠PAQ = ∠PCQ = ∠PAQ = ∠PCQ = $\frac{1}{2}$∠POQ.

The angle in a semi-circle is a right angle.

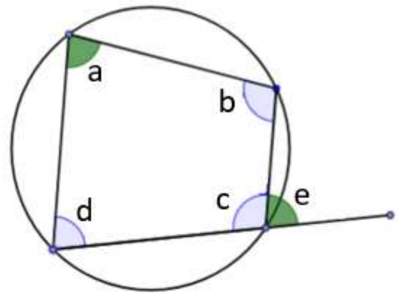
- ∠OAB ∠OAB is an angle subtended by the arc APB at any point C on the semi-circle ACB, then ∠ACB = 90°.



Cyclic Properties of Circle

1. When a quadrilateral is inscribed in a circle i.e. the vertices of the quadrilateral lie on the circumference of a circle; the quadrilateral is called a cyclic quadrilateral.
2. The points, which lie on the circumference of the same circle are called concyclic points.

Theorems related to a Cyclic Quadrilateral



1. If ABCD is a cyclic quadrilateral then:
   1. ∠BAD + ∠BCD = 180°
   2. ∠ABC + ∠ADC = 180°
2. If in any quadrilateral ABCD,
   1. ∠BAD + ∠BCD = 180°
   2. ∠ABC + ∠ADC = 180°

 then ABCD is a cyclic quadrilateral.

Corollary

If a side of a cyclic quadrilateral is produced, the exterior angle is equal to the interior opposite angle.

- Side AB of a cyclic quadrilateral is produced to E.