

Comparing Quantities

Percentage:-A percentage is a number or ratio expressed as a fraction of 100.

It is often denoted using the percent sign, "%", or the abbreviations "pct.", "pct"; sometimes the abbreviation "pc" is also used.

Ratios:-Ratio means comparing two quantities.

For example, a basket has two types of fruits, say, 20 apples and 5 oranges

Thus, the ratio of the number of oranges to the number of apples = 5 : 20

The comparison can be done by using fractions as, $5 / 20 = 1 / 4$

percentage of oranges $5 / 25 \times 100 = 20\%$

percentage of apples $20 / 25 \times 100 = 80\%$

The total sum of individual percentages is always equal to $100 = 20 + 80$.

Increase percent:-

$$\% \text{ increase} = \frac{\text{Final} - \text{Initial}}{\text{Initial}} \times 100.$$

Example: New or final price of an apple = Rs 25

Old or initial price of an apple = Rs 20

$$\text{Thus } \% \text{ increase} = \frac{25 - 20}{20} \times 100 = 25\%$$

Decrease percent:-

$$\% \text{ decrease} = \frac{\text{Initial} - \text{Final}}{\text{Initial}} \times 100.$$

Example: New or final price of an apple = Rs 20

Old or initial price of an apple = Rs 25

$$\text{Thus } \% \text{ decrease} = \frac{25 - 20}{25} \times 100 = 20\%$$

Profit and Loss:-

Important terms:

Cost Price: It is the price at which an article is purchased. It is denoted as C.P.

Selling Price: It is the price at which an article is sold. It is denoted as S.P.

Profit: It is made when the selling price is more than the cost price. It is denoted as P.

Loss: It occurs when the selling price is less than the cost price. It is denoted as L.

Overheads: They are expenses like transportation, commission, repair charges, labour charges etc. which are occurred while buying, thereby increasing the cost price of the article.

Important Formulas:

$$\mathbf{C.P = S.P - Profit}$$

$$\mathbf{C.P = S.P + Loss}$$

$$\mathbf{S.P = C.P + Profit}$$

$$\mathbf{S.P = C.P - Loss}$$

$$\mathbf{Profit (P) = S.P - C.P}$$

$$\mathbf{Loss (L) = C.P - S.P}$$

$$\mathbf{Profit \% = \frac{profit}{C.P} \times 100}$$

$$\mathbf{Loss \% = \frac{Loss}{C.P} \times 100}$$

$$\mathbf{Discount (D) = \text{marked price} - \text{Selling price}}$$

$$\mathbf{Discount \% = \frac{D}{M.P} \times 100}$$

$$\mathbf{Net price = S.P = M.P - D}$$

Sales Tax:- Sales tax is what we pay when we buy items. This sales tax is charged by the government on the sale of an item.

It is collected by the shopkeeper from the customer and given to the government.

This is always on the selling price of an item and is added to the value of the bill.

Bill amount = Cost of item + Sales tax.

Formulas:

Value added tax = tax charged – tax paid.

Value added tax = tax % x (S.P - C.P)

Compound Interest:-

Compound interest (or compounding interest) is interest calculated on the initial principal and also on the accumulated interest of previous periods of a deposit or loan. Compound interest can

be thought of as “interest on interest,” and will make a sum grow at a faster rate than simple interest, which is calculated only on the principal amount.

Terms regarding compound interest:-

Final amount:

The interest (I) at the end of a certain period is added to the original sum (P) to get the amount. This amount now becomes the principle for the next period. This process is repeated until the amount for the last period is found which is the Final Amount (A).

Compound Interest:

The difference between the final amount (A) and the original principal (P) is called the compound interest. C.I. = Amount – Principal.

Deducing formula for Compound Interest:-

Let the principal be P, time be n years and rate be R

Now, Simple interest for the first year will be given as: $SI_1 = \frac{P \times R \times T}{100}$

$$\begin{aligned} \text{Amount after the 1}^{\text{st}} \text{ year will be} &= P + SI_1 \\ &= P + \frac{P \times R \times T}{100} \\ &= P\left(1 + \frac{R}{100}\right) = P_2 \end{aligned}$$

Simple Interest for the second year will be given as: $SI_2 = \frac{P_2 \times R \times T}{100}$

$$\begin{aligned} \text{Amount after 2}^{\text{nd}} \text{ year will be} &= P_2 + SI_2 \\ &= P_2 + \frac{P_2 \times R \times T}{100} \\ &= P_2\left(1 + \frac{R}{100}\right) \\ &= P\left(1 + \frac{R}{100}\right)\left(1 + \frac{R}{100}\right) \\ &= P\left(1 + \frac{R}{100}\right)^2 \end{aligned}$$

Now, If we go on calculating further for n years we can deduce that: $A = P\left(1 + \frac{R}{100}\right)^n$

$$C.I = A - P$$

$$= P \left[\left(1 + \frac{R}{100} \right)^n - 1 \right]$$

Compound interest (compounded half-yearly):-

When the compound interest is compounded half yearly it means in one year the interest is calculated twice which means in the formula for calculating the Interest the number of terms/year will be twice as given in the question as well as the rate percentage will be halved of that is given in the question.

$$\text{Formula:- } A = P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2}$$

Other applications of compound interest's formula:-

Calculation of growth:

This can be used for growth of industries

$$\text{Production after } n \text{ years} = \text{Initial (original) Production} \times \left(1 + \frac{R}{100} \right)^n$$

r% = rate of growth in production.

Depreciation:

If the cost of a machine depreciates by r% every year, then its value after n years is

$$\text{Value after } n \text{ years} = \text{Present value} \times \left(1 + \frac{R}{100} \right)^n$$

Calculation of Population:

When the population of a town, city, village, etc., increases at a certain rate per year

$$\text{Population after } n \text{ years} = \text{Present population} \times \left(1 + \frac{R}{100} \right)^n$$

r% = rate of increase in population.