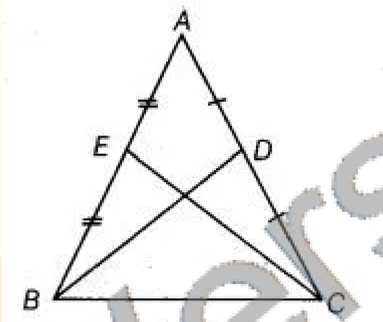
Exercise No. 7.3

Short Answer Questions:

1. ABC is an isosceles triangle with AB = AC and BD and CE are its two medians. Show that BD = CE.

Solution:
Given:
ABC is an isosceles triangle with AB = AC and BD and CE are its two medians.



To prove: BD = CE
Proof: in triangle ABC,
AB=AC [Given]

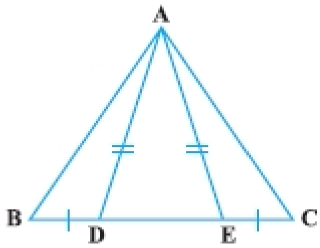
$\frac{1}{2}AB = \frac{1}{2}AC$

AE = AD [D is the mid-point of AC and E is the mid-point of AB]

Now, in triangle ABD and triangle ACE,
AB = AC [Given]
$\angle A = \angle A$ [Common angle]
AE = AD [above proved]

Now, by SAS criterion of congruence, get:
$\Delta ABD \cong \Delta ACE$
BD = CE [CPCT]
Hence, proved.

2. In Fig., D and E are points on side BC of a ΔABC such that BD = CE and AD = AE.
Show that $\Delta ABD \cong \Delta ACE$.

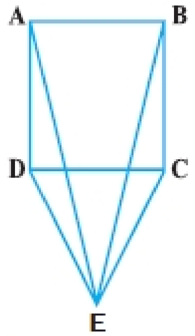


Solution:
Given in triangle ABD,
BD = CE and AD = AE
To prove that $\Delta ABD \cong \Delta ACE$

Proof:
AD = AE [Given]
$\angle ADE = \angle AED$ [Since, angle opposite to equal sides are equal] … (I)
$\angle ADB + \angle ADE = 180°$ [Linear pair axiom]
$\angle ADB = 180° - \angle ADE$
$\angle ADB = 180° - \angle AED$ [From equation (i)]

In triangle ABD and triangle ACE,
$\angle ADB = \angle AEC$ [Since, $\angle AEC + \angle AED = 180°$, linear pair axiom]
BD = CE [Given]
AD = AE [Given]
$\Delta ABD \cong \Delta ACE$ [By SAS congruence rule]

3. CDE is an equilateral triangle formed on a side CD of a square ABCD as shown in fig. Show that $\Delta ADE \cong \Delta BCE$.

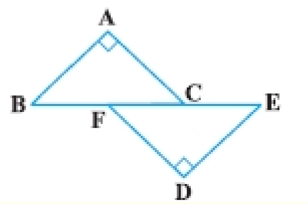


Solution:
Given in figure triangle CDE is an equilateral triangle formed on a side CD of a square ABCD.
To proof that $\Delta ADE \cong \Delta BCE$

Proof: In triangle ADE and triangle BCE,
DE = CE [Sides of an equilateral triangle]
$\angle ADE = \angle BCE$
$\angle ADC = \angle BCD = 90°$ and $\angle EDC = \angle ECD = 60°$

$\angle ADE = 90° + 60° = 150°$ and $\angle BCE = 90° + 60° = 150°$
AD = BC [Sides of a square]
$\Delta ADE \cong \Delta BCE$ [By SAS congruence rule]

4. In Fig., $BA \perp AC$, $DE \perp DF$ such that BA = DE and BF = EC. Show that $\Delta ADC \cong \Delta DEF$.



Solution:
See in the figure,
$BA \perp AC$, $DE \perp DF$ such that BA = DE and BF = EC

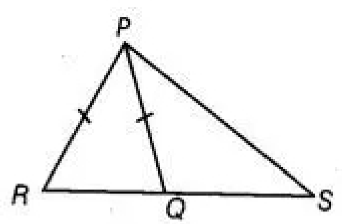
To proof that $\Delta ADC \cong \Delta DEF$
Proof:
BF = EC [Given]

Now, adding CF both sides, get:
$BF + CF = EC + CF$
BC = EF ...(I)

In triangle ABC and triangle DEF,
$\angle A = \angle D = 90°$ [$BA \perp AC$ and $DE \perp DF$]
BC = EF [from eq. (I)]
BA = DE [Given]
$\Delta ABC \cong \Delta DEF$ [By RHS congruence rule]

5. Q is a point on the side SR of a ΔPSR such that PQ = PR. Prove that PS > PQ.

Solution:
In triangle PSR, Q is a point on the side SR such that PQ = PR.
To proof that PS > PQ



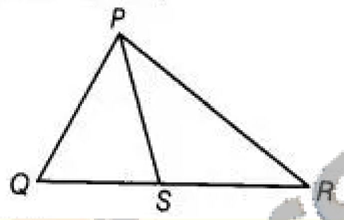
Proof: In triangle PRQ,
PQ = PR [Given]
$\angle R = \angle PQR$ …(I) [Angle opposite to equal sides are equal]

$\angle PQR > \angle S$ … (II) [Exterior angle of a triangle is greater than each of the opposite interior angle]

Now, from equation (I) and (II), get:
$\angle R > \angle S$
PS > PR [side opposite to greater angle is longer]
PS > PQ [PQ = PR]

6. S is any point on side QR of a ΔPQR. Show that: PQ + QR + RP > 2 PS.

Solution:
Given in triangle PQR, S is any point on side QR.



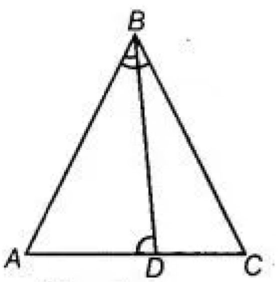
To proof that PQ + QR + RP > 2PS

Proof: In triangle PQS,
PQ + QS > PS (i) [Sum of two side of a triangle is greater than the third side]
Now, similarly in triangle PRS,
SR + RP > PS (ii) [Sum of two side of a triangle is greater than the third side]

Adding equation (I) and (II), get:
PQ + QS + SR + RP > 2PS
PQ +(QS + SR) + RP > 2PS
PQ + QR + RP > 2PS [QR = QS + SR]

7. D is any point on side AC of a ΔABC with AB = AC. Show that CD < BD.

Solution:
Given in triangle ABC, D is any point on side AC such that AB = AC.

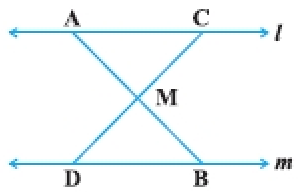


To proof that CD < BD or BD > CD
To proof:
AC = AB [Given]

$\angle ABC = \angle ACB$ (i)[Angle opposite to equal sides are equal]

In triangle ABC and triangle DBC,
$\angle ABC > \angle DBC$ [$\angle DBC$ is a internal angle of $\angle B$]
$\angle ACB > \angle DBC$ [From equation (I)]
BD > CD [Side opposite to greater angle is longer]
CD < BD

8. In Fig., *l* || *m* and M is the mid-point of a line segment AB. Show that M is also the mid-point of any line segment CD, having its end points on *l* and *m*, respectively.

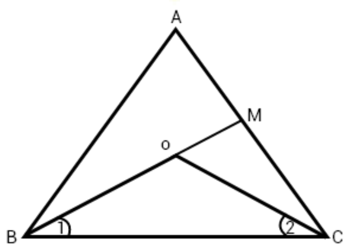


Solution:
See in the figure, $l \parallel m$ and M is the mid-point of a line segment AB.
To proof that MC = MD
Proof: $l \parallel m$ [Given]
$\angle BAC = \angle ABD$ [Alternate interior angles]
$\angle AMC = \angle BMD$ [Vertical opposite angle]

In triangle AMC and triangle BMD,
$\angle BAC = \angle ABD$ [Proved above]
AM = BM [Given]
$\angle AMC = \angle BMD$ [By ASA congruence rule]
MC = MD [By CPCT]

9. Bisectors of the angles B and C of an isosceles triangle with AB = AC intersect each other at O. BO is produced to a point M. Prove that ∠MOC = ∠ABC.

Solution:
Given in the question, bisectors of the angles B and C of an isosceles triangle ABC with AB = AC intersect each other at O. Now BO is produced to a point M.

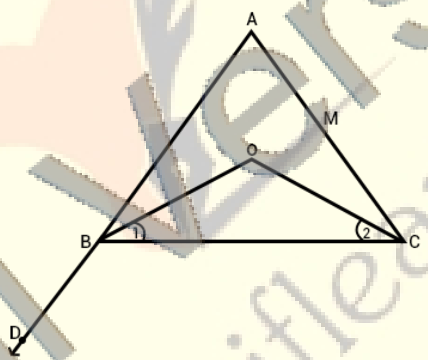


In triangle ABC,
AB = AC
$\angle ABC = \angle ACB$ [Angle opposite to equal sides of a triangle are equal]
$\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

That is $\angle 1 = \angle 2$ [Since, BO and Co are bisectors of $\angle B$ and $\angle C$]
In triangle OBC, Ext. $\angle MOC = \angle 1 + \angle 2$ [Exterior angle of a triangle is equal to the sum of interior opposite angles]
Ext. $\angle MOC = 2\angle 1$ [$\angle 1 = \angle 2$]
Hence, $\angle MOC = \angle ABC$.

10. Bisectors of the angles B and C of an isosceles triangle ABC with AB = AC intersect each other at O. Show that external angle adjacent to ∠ABC is equal to ∠BOC.

Solution:
In triangle ABC,



AB = AC
So, $\angle B = \angle C$ [Angle opposite to equal sides of a triangle are equal]
$\frac{1}{2}\angle B = \frac{1}{2}\angle C$ …(I)

In triangle OBC,
$\angle 1 = \frac{1}{2}\angle B$

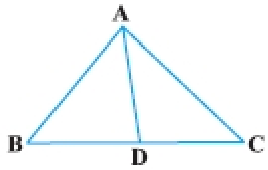
And, $\angle 2 = \frac{1}{2}\angle 2$ [By (I)]
$\angle DBC + \angle 1 + \angle OBA = 180°$ [ABD is a straight line]

In triangle OBC,
$\angle 1 + \angle 2 + \angle BOC = 180°$
$2\angle 1 + \angle BOC = 180°$ [$\angle 1 = \angle 2$]… (II)

From equation (I) and (II), get:
$\angle DBA + 2\angle 1 = 2\angle 1 + \angle BOC$

$\angle DBC = \angle BOC$

11. In Fig. 7.8, AD is the bisector of ∠BAC. Prove that AB > BD.



Solution:
In triangle ACD,
Ext. $\angle ADB > \angle DAC$ [Exterior angle of a triangle is greater than either of the interior opposite angle]
$\angle ADB > \angle BAD$

Since, in triangle ABD,
$\angle ADB > \angle BAD$
Hence, AB>BD. [In a triangle, side opposite to greater angle is longer]